



Comprehensive Curriculum

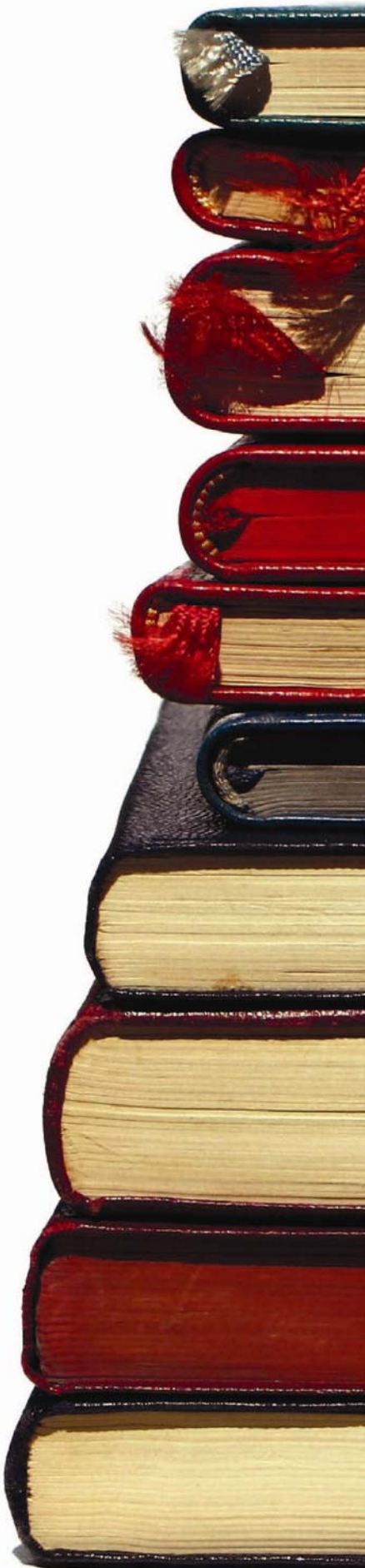
Revised 2008

Grade 7 Mathematics



Louisiana Department of
EDUCATION

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**Grade 7
Mathematics**

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Louisiana Comprehensive Curriculum, Revised 2008 **Course Introduction**

The Louisiana Department of Education issued the *Comprehensive Curriculum* in 2005. The curriculum has been revised based on teacher feedback, an external review by a team of content experts from outside the state, and input from course writers. As in the first edition, the *Louisiana Comprehensive Curriculum*, revised 2008 is aligned with state content standards, as defined by Grade-Level Expectations (GLEs), and organized into coherent, time-bound units with sample activities and classroom assessments to guide teaching and learning. The order of the units ensures that all GLEs to be tested are addressed prior to the administration of *iLEAP* assessments.

District Implementation Guidelines

Local districts are responsible for implementation and monitoring of the *Louisiana Comprehensive Curriculum* and have been delegated the responsibility to decide if

- units are to be taught in the order presented
- substitutions of equivalent activities are allowed
- GLEs can be adequately addressed using fewer activities than presented
- permitted changes are to be made at the district, school, or teacher level

Districts have been requested to inform teachers of decisions made.

Implementation of Activities in the Classroom

Incorporation of activities into lesson plans is critical to the successful implementation of the Louisiana Comprehensive Curriculum. Lesson plans should be designed to introduce students to one or more of the activities, to provide background information and follow-up, and to prepare students for success in mastering the Grade-Level Expectations associated with the activities. Lesson plans should address individual needs of students and should include processes for re-teaching concepts or skills for students who need additional instruction. Appropriate accommodations must be made for students with disabilities.

New Features

Content Area Literacy Strategies are an integral part of approximately one-third of the activities. Strategy names are italicized. The link ([view literacy strategy descriptions](#)) opens a document containing detailed descriptions and examples of the literacy strategies. This document can also be accessed directly at <http://www.louisianaschools.net/1de/uploads/11056.doc>.

A *Materials List* is provided for each activity and *Blackline Masters (BLMs)* are provided to assist in the delivery of activities or to assess student learning. A separate Blackline Master document is provided for each course.

The *Access Guide to the Comprehensive Curriculum* is an online database of suggested strategies, accommodations, assistive technology, and assessment options that may provide greater access to the curriculum activities. The *Access Guide* will be piloted during the 2008-2009 school year in Grades 4 and 8, with other grades to be added over time. Click on the *Access Guide* icon found on the first page of each unit or by going directly to the url <http://mconn.doe.state.la.us/accessguide/default.aspx>.



Grade 7
Mathematics
Unit 1: Fractions, Decimals, and Percents

Time Frame: Approximately four weeks



Unit Description

The focus of this unit is connecting and extending the relationships of fractions, decimals, integers and percents to enable deeper understanding and flexibility in thinking. Proportionality is explored.

Student Understandings

Students demonstrate their grasp of fraction, decimal, integer, and ratio/percent representations and operational understandings by comparing, ordering, contrasting, and connecting these numbers to real-life settings and solving problems. They demonstrate an understanding of reasonableness of answers by comparing them to estimates. Students can distinguish between unit rates and ratios and recognize quantities that are related proportionally.

Guiding Questions

1. Can students represent in equivalent forms and evaluate fractions, percents, decimals, integers, and ratios?
2. Can students connect fractions, decimals, integers, and ratios to their real-life applications?
3. Can students demonstrate the equality of ratios in a proportion?
4. Can students illustrate the reasonableness of answers to such problems?

Unit 1 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
1.	Recognize and compute equivalent representations of fractions, decimals, and percents (i.e., halves, thirds, fourths, fifths, eighths, tenths, hundredths) (N-1-M)
2.	Compare positive fractions, decimals, percents, and integers using symbols (i.e., $<$, \leq , $=$, \geq , $>$) and position on a number line (N-2-M)

GLE #	GLE Text and Benchmarks
6.	Set up and solve simple percent problems using various strategies, including mental math (N-5-M) (N-6-M) (N-8-M)
7.	Select and discuss appropriate operations and solve single- and multi-step, real-life problems involving positive fractions, percents, mixed numbers, decimals, and positive and negative integers (N-5-M) (N-3-M) (N-4-M)
8.	Determine the reasonableness of answers involving positive fractions and decimals by comparing them to estimates (N-6-M) (N-7-M)
9.	Determine when an estimate is sufficient and when an exact answer is needed in real-life problems using decimals and percents (N-7-M) (N-5-M)
10.	Determine and apply rates and ratios (N-8-M)
11.	Use proportions involving whole numbers to solve real-life problems. (N-8-M)

Sample Activities

Activity 1: Decimal Comparisons - Where's the Best Place? (GLE: 2)

Materials List: Where's the Best Place BLM, Numbers BLM , learning log

Students will create numbers in decimal form and write inequalities with them.

Students play a game called Where's the Best Place? Review symbols used to compare numbers ($>$, $<$, \geq , \leq , $=$). Place the students in groups of 4.

Rules for the game:

- Give each player a copy of the Where's the Best Place BLM to play the game.
- Have students shuffle ten cards numbered 0 through 9 and place them face down in a pile. (Use the Numbers BLM to make the cards or use the cards 2-9 and an ace from a deck of playing cards.)
- One player draws a digit card from the pile. Each player must decide privately where he/she wants to write that digit on his/her game card. The object is to try to create the largest number.
- After the player writes a digit on the game card, he/she cannot erase it and place it elsewhere. Once a digit is drawn, it cannot be used again in that game.
- The game is over when all places on each game card are filled. The player with the greatest number wins.
- Write an inequality using the four numbers generated by the group.

Students should respond to the following prompt in their *learning logs* ([view literacy strategy descriptions](#)). This *learning log* should be a small notebook used primarily for recording math understanding. Explain to the students that their *learning logs* will be

used all year to record new learning and write questions that they want answered through math class. Have students copy the prompt. For longer prompts, the teacher should copy the prompt and have students tape, glue, or staple into the *learning log*.

Prompt:

Some of the digits in the following numbers are hidden.

A. $3.\overline{\text{■}}\overline{\text{■}}\overline{\text{■}}$

B. $3.\overline{\text{■}}\overline{\text{■}}$

Give an example when each situation is true. Using mathematics, justify your answers.

1. the value of A is larger than the value of B
2. the value of B is larger than the value of A
3. the value of A is equal to the value of B

Activity 2: Fraction Comparisons (GLEs: 1, 2)

Materials List: several pieces of chart paper for every pair of students, Fraction Comparisons BLM for each pair of students

Students will write equivalent and nonequivalent fractions as well as inequalities to compare them.

To check the depth of understanding students have in dealing with equivalent fractions, have the students complete the Fraction Comparisons BLM while working with a partner. Circulate around the room and ask questions to find what strategies the students are using to find equivalent fractions.

A copy of the information provided on the BLM is reprinted below.

1. Using chart paper, complete the following situation. Be prepared to share your work in 20 minutes.
 - a. Write two fractions that are equivalent. Explain how you know that they are equivalent.
 - b. Look at the fractions you wrote in part a. Write two other fractions, one that is equivalent to your first fraction and one that is equivalent to the second fraction.
 - c. Are the four fractions you have written equivalent to each other? Why or why not?
2. Using chart paper, complete the following situation. Be prepared to share your work in 20 minutes.
 - a. Write two fractions that are not equivalent. Tell which is larger, and explain how you know.
 - b. Look at the fraction you wrote in part a. Write two other fractions, one that is **not** equivalent to your first fraction and another one that is **not** equivalent to your second fraction.

- c. Order the four fractions you have written from smallest to largest, and explain how you know the order is correct.
- d. Write a mathematical statement using the symbols $<$, \leq , $=$, \geq , $>$ and your fractions.

Activity 3: Number Line Placement (GLEs: 1, 2)

Materials List: *Velcro*[®] strip, masking tape, or string for number line; a set of rational number index cards

Use this activity as a pre-assessment activity to get an idea of the students' level of understanding of number sense.

Use a *Velcro*[®] strip, masking tape, or string taped along the board to represent a number line. Place zero and one on the number line. Have students compare and determine the placement of rational numbers. Have numbers written on index cards for the students to use. (Examples of numbers: 1, $\frac{1}{2}$, 100%, .08, $\frac{1}{3}$, 75%, 0) Make sure to include several numbers which are equivalent—fractions, decimals and percents. Do not use negative integers at this time. Give a card to a student. Have him/her place the card where he/she thinks it belongs on the number line using masking tape or a *Velcro*[®] strip. Have a discussion about the placement of this number (e.g., Must it go there? Could it be placed elsewhere?). Give another card for placement to another student. Continue until all numbers have been placed along the number line. There are many questions that can be asked with the placement of each number creating in-depth class discussions. Students may need to move some numbers on the number line once one or two numbers have been placed. Have students make observations about the number line and write 5 inequalities from the number line using the symbols $<$, \leq , $=$, \geq , $>$.

Activity 4: Representation of Equivalent Fractions, Decimals, and Percents (GLE: 1)

Materials List: Fraction Pieces BLMs (eight BLMs) for each student, scissors

Give each student a copy of each of the eight Fraction Pieces BLMs. Each BLM should be copied on a different color of paper. Each sheet will have a rectangle divided into equal portions by parallel lines. The rectangle on Fraction Pieces 2 BLM is divided into halves. The rectangle on Fraction Pieces 3 BLM is divided into fourths, etc. Model labeling and cutting strips from the paper using colored overhead sheets (i.e., cut along the parallel lines and then at the marking for $\frac{1}{2}$). Show students how to represent each fraction, decimal, and percent with a different colored paper. A red strip of paper is cut into 2 pieces and each piece is labeled $\frac{1}{2}$, 0.50, and 50%. A blue strip of paper is cut into 4 pieces and each piece is labeled $\frac{1}{4}$, 0.25, and 25%, etc. On the overhead, show the placement of equivalent fractions of two different colors (e.g., 1 red piece is equivalent to

2 blue pieces, shown side by side on the overhead). Lead a discussion which includes equivalencies using decimals and percents. Have students work in groups of 4, cut their papers into pieces as modeled, and develop a presentation showing the maximum number of equivalent fractions.

Teacher Notes:

1. *Caution, if one graphic is resized, all graphics will need to be resized proportionally. Always copy the original. Copy and compare the sizes of each strip before duplicating in mass to give to students. Many photocopiers do not make exact duplicates of an image and the more copies that are made, the more variance there could be. The heat expands the paper. The last page copied may be more distorted than the first if the machine is hot.*

2. *Fraction pieces are also used in Unit 1, Activity 5 and Unit 2, Activity 1. Have each group store the pieces in a gallon baggie to for future use.*

Activity 5: Compare Fractions, Decimals, and Percents (GLE: 2)

Materials List: fraction strips from Activity 4, 6 index cards for each student (two fractions, two decimals, and 2 percents); Greater Than, Less Than, or Equal To BLM for each student or pair of students.

Review the concepts of equal, greater than, and less than with students who will work in groups of 2. Using the colored overhead strips from Activity 4, demonstrate the concepts of comparing numbers using the terms *greater than*, *less than*, *greater than or equal to*, *less than or equal to* and *equal*.

Give 6 index cards that include two fractions, two decimals, and two percents to each student. Have students form pairs and instruct each student to randomly select an index card. The pair should write an equality using the numbers on the cards drawn and be able to defend their reasoning to others. After several minutes, instruct groups to switch inequalities with another group and check each other’s work.

Have each pair of students complete the modified math *word grid* ([view literacy strategy descriptions](#)) found on the Greater Than, Less Than, or Equal To BLM to show their understanding of greater than, less than, greater than or equal to, less than or equal to, and equal. (Students should use the values given to them earlier to fill in the left column. Students may also create new fractions by rolling a number cube.)

Example

	Greater Than, Less Than, or Equal To				
	$> \frac{1}{2}$	$\leq \frac{1}{2}$	$= \frac{1}{2}$	$> 20\%$	< 0.75
$\frac{1}{4}$		✓		✓	✓
50%		✓	✓	✓	✓

Place a check in any cell to indicate which statements are true when the number in the first column is combined with the information in the top row.

Activity 6: Equivalent Fractions, Decimals, and Percents (GLEs: 1, 2)

Materials List: at least 30 index cards per 4 students

Have groups of four students create a deck of cards using index cards. Cards should represent common fractions such as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{3}$, $\frac{2}{5}$, etc. and their decimal and percent equivalencies. Example: one card will have 0.5, a second card will have $\frac{1}{2}$, and the third card 50%. The three equivalent cards represent a set. Each deck of cards should contain 10 complete sets. A game is played in which five cards are dealt to each player and the rest are laid down for a draw. Use the rules for a Go Fish game. When a student draws a card, he/she asks, "Do you have anything equal to ____?" (e.g., Do you have anything equal to $\frac{1}{2}$? Do you have anything equal to 20%?). The students lay down cards when they have all three cards which comprise a set. The first student to use all of his/her cards wins.

Using the cards created, reinforce the concept of greater than, less than, and equal to, greater than or equal to and less than or equal to. Create cards for each of these symbols. Divide students into teams to play a spelling bee type game in which two cards are drawn from the deck of fractions, decimals, and percents. The team drawing the cards has one minute to choose which symbol is appropriate and explain why they chose the inequality symbol. If they cannot, the other team is given a chance. Scoring is one point per correct answer.

Activity 7: Is it Reasonable? (GLEs: 7, 8)

Materials List: teacher-made set of real-life problems involving positive fractions and decimals, paper, pencil, math learning log

Provide students with a list of real-life situations involving positive fractions and decimals. Individually, have the students estimate each answer. As a class, discuss their estimates and methods used for estimating. Example problem: 24% of the 7th graders at West Middle School are helping tutor 4th graders at West Elementary School. If there are 322 seventh graders at West Middle School, estimate how many seventh grade students are tutoring the 4th graders.

Give the students a list of the correct answers, and have them select the appropriate exact answer from the list. Discuss the operations needed to solve the problems. Ask the students to compare their estimations to the exact answers. Were any estimations way off? Have a discussion of how far away from the correct answer is too far. Be sure to point out there is no 'set limit'; it depends on the information. Give students examples such as this: when estimating the number of students in a classroom, ten students make a big difference, but if you are talking about estimating the number of people at a concert, ten people would not make a difference. Discuss what makes one estimate better than another?

Students should respond to the following prompt in their math *learning log* ([view literacy strategy descriptions](#)).

Prompt:

Pam's class is asked to estimate 5.3% of 41.9. Pam estimates 8. Keith estimates 20, and Seth estimates 2. Who has the best estimate? Justify your answer using words and mathematical symbols.

Activity 8: What Is Needed? (GLE: 9)

Materials List: pencil, paper

In groups of four, have students *brainstorm* ([view literacy strategy descriptions](#)) to develop a list of scenarios that require exact answers to problems involving decimals and percents and a list of scenarios in which an estimate is appropriate. For example, if a person were shopping for groceries and had \$30 in cash, then an estimate of the cost of items in a grocery cart can be used to determine if some of the items should be put back. Have groups write word problem for one scenario that requires an estimate and for one that requires an exact answer and share them with the class. Groups can exchange problems with another group, solve the problems, then return them to the original group for checking.

Activity 9: Simple Percent Problems (GLEs: 6, 8)

Materials List: newspaper advertisements or B's Shoe Boutique BLM (at least one example for each group of 4 students), a half or quarter sheet of poster board per group, glue, scissors, markers/colored pencils

Divide students into groups of four. Introduce the idea of shopping when a store is having a sale. Using the store sale advertisements from the newspaper or B's Shoe Boutique BLM, have the student groups figure 10%, 20%, 30%, 50%, 75%, $33\frac{1}{3}\%$ and $66\frac{2}{3}\%$ off the cost of items in the advertisements, or figure the sale price using the percent that is given in the ad. Many times items are advertised as $\frac{1}{3}$ or $\frac{2}{3}$ off the original price. Make sure students know that these are the same as $33\frac{1}{3}\%$ and $66\frac{2}{3}\%$ off.

Have the students check to see if their answers are reasonable. Have students practice estimating 10%, 20%, 30%, 50%, 75%, $33\frac{1}{3}\%$ and $66\frac{2}{3}\%$ off the items in the ads, and then compare these answers to the answers they originally figured.

Give each group a budget and assign different discounts. Have students choose items from the sale papers, estimate the percents to determine if they have enough money to make the purchases they want, and then calculate the exact prices. On a quarter/half sheet of poster board, have the students create a display indicating their choices, the method used to calculate each price, and the total cost of their purchases. Allow students to cut

and paste pictures of the items, and require them to show their work. As an extension, have students add the local sales tax or create a grocery shopping scenario.

Activity 10: Tipping at a Restaurant (GLEs: 6, 8,)

Materials List: Tipping at a Restaurant BLM for each student or group, pencil

Discuss with the class the tip customers leave at restaurants, noting that customers pay their server a tip for providing good service. A typical tip is 15% to 20% of the cost of the meal. Indicate to students that they need to use estimation skill to figure a tip that will be left for the server because the check will seldom be a whole number. Discuss with the students how to round in reasonable ways. Discuss mental math strategies when finding the tip at a restaurant.

Present the following situation to the class. Your bill at Logan's Restaurant is \$19.45. What is a 10% tip on this bill? Instruct students to round off the amount to something they can reasonably work with. Some may say \$19.50, but ask if this is reasonable for the situation. They may then say \$1.95. Would it be more reasonable to leave \$1.95 or \$2.00? So a better process might be to round \$19.45 up to \$20.00 and then calculate a 10% tip for \$20.00. Then have the students calculate 10% of \$19.45. Have students compare their estimate with the calculation and check for reasonableness.

Practice several different amounts where students will need to use estimation and rounding to get a 10%, 15%, and 20% tip. Stress techniques that apply the distributive property: 15% is a 10% tip plus half that amount, 20% is double a 10% tip. Additional scenarios may be found on Tipping at a Restaurant BLM.

Activity 11: Rates (GLE: 10)

Materials List: sale papers/grocery items that can be used to figure unit cost and/or a copy of Grocery Shopping BLM, pencil, paper

Provide the students with a list of items they can purchase along with the prices. These items can be 6-packs of soft drinks, ounces of potato chips, pounds of peanuts, and so on. Be sure to use items that can be used to figure unit cost. Have the students calculate the unit price of each item. Also, extend this to include rates such as \$45.00 for 8 hours of work, driving 297 miles in 5 hours, reading 36 pages in 2 hours, and so on. Have the students figure unit rates for these types of problems also.

Put students in small groups, and give the students 5-10 minutes to review the information from the activity and to respond to one or both of the following situations. They should also write at least 3-5 questions they anticipate being asked by their peers and 2-5 questions to ask other experts. When time is up, the teacher will randomly select groups to assume the role of *professor know-it-all* ([view literacy strategy descriptions](#))

and provide their answers and reasoning for the situation. They will also have to provide “expert” answers to questions from their peers about their reasoning.

Situation 1: Lucy and CJ are in charge of buying chips for a class party. They plan to purchase 1.5 to 2 oz of chips for each of the 24 students. Use the information below to help them make the best purchase.

Big Al’s Grocery
 1 – 1.75oz can for \$0.75
 12 – 1.75oz cans for \$8.95
 1 – 6oz can for \$2.52

Solution:

<i>Size</i>	<i>Total Ounces</i>	<i>Total Cost</i>	<i>Cost per Ounce</i>	
<i>1 – 1.75 oz can</i>	<i>1.75</i>	<i>\$0.75</i>	<i>0.428</i>	
<i>12 – 1.75 oz cans</i>	<i>21</i>	<i>\$8.95</i>	<i>0.426</i>	
<i>1 – 6 oz can</i>	<i>6</i>	<i>\$2.55</i>	<i>0.425</i>	<i>X</i>

Situation 2: Kenneth and Jena are in charge of buying sodas for a class party. They plan to purchase 6 oz of soda for each of the 26 students. Use the information in the table to help them make the best purchase.

PJ’s Grocery

<i>Container Size</i>	<i>Capacity in ounces</i>	<i>Cost</i>
<i>1 Liter</i>	<i>33.8 oz</i>	<i>\$1.09</i>
<i>2 Liter</i>	<i>67.6 oz</i>	<i>\$1.29</i>
<i>3 Liter</i>	<i>101.4 oz</i>	<i>\$1.99</i>

Solution: This table shows the different combinations of containers the students may use to get their target of 156 ounces. Be sure students are able to defend their choices; they may not choose the overall lowest unit cost which requires them to purchase additional soda. They will need to purchase a minimum of 156 ounces of soda.

<i>Quantity</i>	<i>Size</i>	<i>Total Ounces</i>	<i>Total Cost</i>	<i>Cost per Ounce</i>	
<i>5</i>	<i>1-L</i>	<i>169 oz</i>	<i>\$5.45</i>	<i>\$0.03</i>	
<i>2</i>	<i>2-L</i>				
<i>1</i>	<i>1-L</i>	<i>169 oz</i>	<i>\$3.67</i>	<i>\$0.021</i>	
<i>3</i>	<i>2-L</i>	<i>202.8 oz</i>	<i>\$3.87</i>	<i>\$0.0190</i>	<i>X</i>
<i>1</i>	<i>3-L</i>				
<i>1</i>	<i>2-L</i>	<i>169 oz</i>	<i>\$3.28</i>	<i>\$0.0194</i>	
<i>2</i>	<i>3-L</i>	<i>202.8 oz</i>	<i>\$3.98</i>	<i>\$0.0196</i>	
<i>1</i>	<i>3-L</i>				
<i>2</i>	<i>1-L</i>	<i>169 oz</i>	<i>\$4.17</i>	<i>\$0.024</i>	

Activity 12: Ratio Patterns (GLEs: 10, 11)

Materials List: pattern blocks or pieces of paper in 5 colors with squares, rectangles and triangles, scissors, pictures of quilts, patterns, repeating patterns

Show the class pictures of quilts and patterns that have a repeating pattern such as an AB, ABA, or ABC pattern, and pass out five different colors of paper (pattern blocks, if available) marked with varying shapes including squares, rectangles, and triangles about 2 inches in size. Divide students into groups, and have them cut out the shapes. Show students two shapes - an equal number of red squares and blue triangles. Discuss the ratio of red pieces to blue pieces or squares to triangles. Group different color pieces and shapes to create designs. Discuss how repeated patterns are pleasing to the eye. Ask volunteers to come forward to create a pattern with pieces. Have the volunteers give the ratio of the colors or shapes. Divide students into groups to create their own patterns using different color ratios. Next, give each group a different ratio of reds to greens and blues to yellow, etc. (e.g., the ratio of 3 blue to every 4 green or 2 red for every 5 yellow) and have students create a pattern and demonstrate how their ratio was used to create the pattern. Introduce the concept of *proportion* for the patterns the students have created (e.g. 4 green for every 2 red is the same as 8 green for every 4 red). Demonstrate how to set up a proportion: $\frac{4}{2} = \frac{8}{4}$. Help students realize the two fractions are equivalent; the second has only increased by a common factor of 2. Cross multiply to create an equation that shows the cross products are equal: $4 \times 4 = 2 \times 8$. Instruct students how to solve the equation; in this example it is just a matter of simplifying each side. Now tell the students the ratio of yellow to red is 3 to 4. If I have 20 red I want to use, how many yellow will I need? Again, demonstrate how to set up and solve a proportion to find the solution. Vary the difficulty of the task by specifying the pattern (e.g., the same colors cannot touch on more than two sides).

Challenge the students to create their own quilt pattern and to determine the ratio/proportion of the colors they used. Have the students change the ratio of the colors used to a percent of colors used.

Activity 13: What's the Recipe? (GLEs: 7, 11)

Materials List: a different recipe for each pair of students, What's the Recipe BLM

Discuss situations where a recipe may need to be reduced or increased. Discuss the fact that all ingredients must be increased or decreased *proportionally* in order for the recipe to turn out correctly. (For example, use a recipe for making chocolate chip cookies that makes 24 cookies, but the recipe needs to be increased so that everyone in a class of 36 gets a cookie.) If the given recipe produces 2 dozen cookies, what would the recipe be for producing 1 dozen cookies? 4 dozen? 6 dozen? 7 dozen? $\frac{1}{2}$ dozen? Give each pair of students a different recipe or the What's the Recipe BLM, and have them reduce or

increase the recipe proportionally by two given amounts. Also, have students create new recipes based on a given percent of the original recipe.

A hot chocolate recipe is a good choice; after calculating how much of each ingredient, the students could make hot chocolate for the class. Water could be heated in a coffee pot.

Activity 14: Number Line with Integers (GLE: 2)

Materials List: *Velcro*[®] strip or string and tape for number line (from Activity 3), a set of index cards labeled with positive and negative integers

Using a *Velcro*[®] strip or string number line from Activity 3, have students compare rational number and integers and determine the placement of the numbers on the number line. Make index cards for the students to use. (Examples of numbers: 1, -1, $\frac{1}{2}$, 100%, -4, $-\frac{1}{3}$, 50%, 0). Make sure to use positive and negative fractions and decimals, whole numbers, and percents to place on the number line. Give a card to a student and ask him/her to place the card where it belongs on the number line. Have a discussion about the placement of this number. Must it go in that exact location? Could it be placed elsewhere? Give out another card for placement. Continue until all numbers have been placed along the number line. Ask many questions about the placement of each number creating in-depth class discussions. Some numbers may need to be moved on the number line once one or two numbers have been placed on the number line.

Sample Assessments

General Assessments:

- Determine student understanding as the student engages in the various activities.
- Whenever possible, create extensions to an activity by increasing the difficulty or by asking “what if” questions.
- Encourage students to create their own questions.
- Ask students to create and demonstrate math problems by acting them out or using manipulatives to provide solutions on the board or overhead.
- Observe the student’s presentations.
- Have students complete math *learning log* entries ([view literacy strategy descriptions](#)) by responding to prompts such as:
 - Explain the meaning of 10%, 20%, 25%, $33\frac{1}{3}\%$, 50%, $66\frac{2}{3}\%$, 75%, and 100% and write their fractional and decimal equivalents. Give examples of their use in real-life situations.

- The 4-H advisor is in charge of buying drinks for the club's landscaping day. She conducted a survey to determine if students liked *Dr. Pepper*[®] or *Coca Cola*[®]. Here are her results:

	Grade 6	Grade 7	Grade 8
<i>Dr. Pepper</i> [®]	80	75	85
<i>Coca Cola</i> [®]	70	90	80

Tell whether the statements 1-4 are accurate based on the information in the table. Explain your answer for each item.

- 15 more seventh graders prefer *Dr. Pepper*[®] to *Coca Cola*[®].
- The ratio of seventh graders who prefer *Dr. Pepper*[®] to *Coca Cola*[®] is 5 to 6.
- 50% of the students surveyed prefer *Dr. Pepper*[®].
- $\frac{7}{8}$ of the sixth graders prefer *Coca Cola*[®].

Tell whether an exact answer or an estimate is needed to determine the grade in which 52% preferred *Dr. Pepper*[®]. Explain your answer.

The *Coca Cola*[®] for the party would cost \$168 and the *Dr. Pepper*[®] would cost \$180. Will the students pay the same price for *Coca Cola*[®] and *Dr. Pepper*[®]? Justify your answer using the cost per student.

Teacher may want to copy longer prompts and have students tape, glue, or staple into math learning logs.

- Assign the following project: Collect several flyers from local restaurants advertising their specials and menu items. In groups of four, students will plan a dinner party at a restaurant for their group with a set budget and prepare a presentation on poster board. The poster will show each person's order, tax and tip on the total bill, and the final cost.

Activity-Specific Assessments:

- Activity 3: Given a list of 8 different representation of numbers (fractions, decimals, percents) and a blank number line, the student will place the numbers in the correct position on the number line and write three inequalities using the given numbers and the symbols $<$, \leq , \geq , $>$.

- **Activity 7:** Present the following scenario to the student, and evaluate the student's ability to answer the questions asked orally.
Latoya is at a grocery store near her house. She has \$10.00, but no calculator or paper or pencil. At the right is a list of the items she would like to buy. Use mental calculations and estimation to answer the following questions.

Item	Price
Milk	\$2.47
Eggs	\$1.09
Cheese	\$1.95
Bread	\$0.68
Honey	\$1.19
Cereal	\$3.25
Avocado	\$0.50

1. Latoya believes she can purchase all of the items she wants. Is this reasonable? Justify your answer.
2. What different items could she buy to come as close as possible to spending \$5.00?
3. Approximately what percent of the \$10.00 did Latoya spend on eggs?

Solutions:

1. No, she cannot buy all the items. By estimating to the nearest half dollar, we know she will need at least \$10.50. She must have \$11.13 before tax.

2. Solutions may vary: Sample solutions: milk, avocado, and cheese or eggs, cheese, honey and avocado

3. \$1.09 out of \$10.00 is about 10%.

- **Activity 9:** On a sheet of unlined paper, the student will create an ad for the newspaper. The ad must include the item (a drawn picture) with a description, the regular price of the item, the percent of discount, and sale price. The student will show how he/she arrived at the sale price on the back of the ad.

- **Activity 13:** The student will work the following problem correctly: A certain recipe for brownies calls for 2 teaspoons of vanilla and 6 teaspoons of oil. If you want to make a large batch of brownies for your class using 10 teaspoons of oil, how much vanilla would you need? Hint--make a table.

Solution: $3\frac{1}{3}$ teaspoons of vanilla

oil	3	6	9	10	11	12
vanilla	1	2	3	$3\frac{1}{3}$	$3\frac{2}{3}$	4

Grade 7
Mathematics
Unit 2: Computation with Fractions, Decimals, and Proportions

Time Frame: Approximately four weeks



Unit Description

This unit extends the work of the previous unit to include the operational understandings of multiplication and division of fractions and decimals and their connections to real-life situations including using ratios and rates. Extending order of operation situations to higher levels will also be addressed.

Student Understandings

Students develop an understanding of multiplication and division of fractions and decimals using concrete models and representations. At the same time, they will become proficient in computations that involve positive fractions, mixed numbers, decimals, and positive and negative integers using the order of operations. Students also develop an overall grasp for solving proportions involving whole numbers. Students should distinguish between rates and ratios, and set-up, analyze, and explain methods for solving proportions. Students should be able to distinguish between situations where an estimate is sufficient and where an exact answer is needed.

Guiding Questions

1. Can students multiply and divide fractions and decimals with understanding of the operations and accompanying representations?
2. Can students add, subtract, multiply, and divide negative integers?
3. Can students set up and solve proportions involving whole number solutions?
4. Can students interpret the results of operations and their representations, for example, between ratios and rates?
5. Can students determine when computations are required or just estimates in real-life settings?
6. Can students tell if answers to operations are reasonable?

Unit 2 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
3.	Solve order of operations problems involving grouping symbols and multiple

GLE #	GLE Text and Benchmarks
	operations (N-4-M)
5.	Multiply and divide positive fractions and decimals (N-5-M)
7.	Select and discuss appropriate operations and solve single- and multi-step, real-life problems involving positive fractions, percents, mixed numbers, decimals, and positive and negative integers (N-5-M) (N-3-M) (N-4-M)
8.	Determine the reasonableness of answers involving positive fractions and decimals by comparing them to estimates (N-6-M) (N-7-M)
9.	Determine when an estimate is sufficient and when an exact answer is needed in real-life problems using decimals and percents (N-7-M) (N-5-M)
10.	Determine and apply rates and ratios (N-8-M)
11.	Use proportions involving whole numbers to solve real-life problems (N-8-M)
Patterns, Relations, and Functions	
40.	Analyze and verbally describe real-life additive and multiplicative patterns involving fractions and integers (P-1-M) (P-4-M)

Sample Activities

Activity 1: Patterns with Fractions, Decimals and Percents (GLE: 40)

Materials List: Fraction Pieces 9 BLM (one set for each group of students), Patterns BLM copied on overhead sheets, overhead, scissors, pencil, math learning log

This activity begins the same way as Unit 1, Activity 4. Use the same colors for the BLMs used in that activity, but use a different color for the Fraction Pieces 9, BLM. If the pieces from the previous activity were saved, then adjust the student directions to include cutting of only the new sheet and the labeling of those pieces.

Give eight different colored sheets of paper to each group of four students. Each sheet will be printed with parallel lines dividing them into halves, fourths, sixths, eighths, twelfths, sixteenths, twenty-fourths, and thirds (e.g., red is halves, blue is fourths, green is sixths, purple is eighths.) Model the labeling and cutting of the pieces using colored overhead sheets. Represent each fraction, decimal, and percent with a different color of paper. A red piece of paper is cut into 2 pieces and each piece is labeled $\frac{1}{2}$, 0.50, and 50%. A blue piece of paper is cut into 4 pieces and each piece is labeled $\frac{1}{4}$, 0.25, and 25%, etc. (The color of each sheet is not important as long as each sheet is a different color and the same color it was in Unit 1, Activity 4.)

The objective is to have students find the patterns of multiplication of the fractions using the pieces. One blue piece is equivalent to half of a red piece which is $\frac{1}{4}$. Have students model this at their desks. Ask a student to model this situation on the overhead and then write the problem: $\frac{1}{4} = \frac{1}{2}$ of $\frac{1}{2}$ or $\frac{1}{4} = \frac{1}{2} \times \frac{1}{2}$ on the board. Using the different fraction strips,

have the students discover a pattern for the multiplication of these fractions.

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}, \quad \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}, \quad \frac{1}{2} \times \frac{1}{8} = \frac{1}{16}, \quad \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}, \quad \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}, \quad \frac{1}{2} \times \frac{1}{12} = \frac{1}{24}$$

Ask, “Can anyone think of any other fractions that would follow this pattern?” Be sure to have a discussion about decimals and percents, and show how they relate to this situation.

Have students record what they discovered about multiplying fractions in their *learning logs* ([view literacy strategy descriptions](#)). They should also include a sketch of at least one model that justifies their thoughts.

Activity 2: The Meaning of Multiplication of Fractions (GLEs: 5, 8, 40)

Materials List: pencils, paper, a piece of newsprint or similar paper for each pair of students, markers

Ensure that students get a real sense of multiplying fractions and making the connection to the meaning of multiplication.

Ask the students to illustrate the meaning of 3×4 using a picture and/or words. The students should write in words and model three groups of four and/or four groups of three. Make sure the students understand they are adding 3 groups of 4 or 4 groups of 3. This is a good place to review the commutative property. Have a class discussion about a real-life meaning of this problem (e.g., Sam has three groups of candy bars with four candy bars in each group). Extend this concept to include multiplication of a fraction and a whole number (e.g., $3 \times \frac{1}{2}$ *add three groups of one half*). Discuss how to first estimate an answer. This will provide something to compare to the product so students can make sure their answers are reasonable. Ask, “If you multiply a positive number by a positive fraction less than one, will the product be greater than, less than or equal to the first factor?” Write each problem on the board, and ask a student to model it for the class.

(e.g., add three groups of $\frac{1}{2}$ and/or find $\frac{1}{2}$ group of 3 and drawing the groups). Remind students to check for reasonable answers/models.

After doing several of these types of problems, ask the students to create a rule for multiplying whole numbers and fractions. Continue practicing and modeling various situations - fractions times fractions, then fractions and mixed numerals. When discussing a whole number times a mixed number, introduce the concept of the distributive property. $2 \times 4 \frac{1}{2}$ means add two groups of four and a half but also could be written as add two groups of four; add two groups of one half, and then add the two sums. Each time the students model and explain their answers, have them check to see if the answers are reasonable.

Professor Know-It-All ([view literacy strategy descriptions](#)) Have the students work in pairs to create a word problem that involves multiplication of fractions, whole numbers, and mixed numbers. Each group should create/illustrate a model of the problem, write a

mathematical sentence that illustrates the situation, and solve their problem. They should also write at least 3-5 questions they anticipate being asked by their peers and 2-5 questions to ask other experts. Remind students they must be ready to defend the reasonableness of their problems, thought processes, and solutions to the class. After students have been given time to complete their problems, choose groups at random to assume the role of *professor know-it-all*.

Information about and examples of the commutative property and distributive property can be found at Purple Math.com, <http://www.purplemath.com/modules/numbprop.htm>.

Activity 3: Multiplication of Fraction Using Arrays (GLEs: 5, 8, 40)

Materials List: grid paper, pencils, colored pencils, or markers, learning log, Multiplying Fractions BLM

Have a discussion of the meaning of multiplication of whole numbers (e.g., 3×4) using arrays. Give students grid paper and have them create an array that could be used to solve the problem 3×4 .

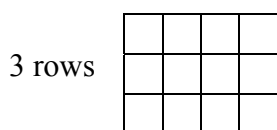
Check to see that students make these drawings and have these understandings.

I have an array of 4 columns
with 3 rows in each column.

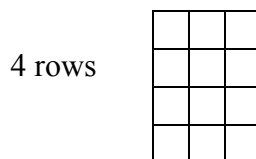
or

I have an array of 3 columns with 4 rows
in each column

4 columns



3 columns



Either way the array is arranged, there are still 12 boxes in the array. The students may use the commutative property to illustrate their arrays if it seems easier for them.

Ask students if they can use arrays in the same way to model multiplication of fractions. In groups, have the students use grid paper to model the situation: Jacque wants $\frac{4}{5}$ of $\frac{1}{2}$ of Nick's candy bar. How much of the whole candy bar does Jacque get? A student, a group of students, or the teacher should model the problem on the board or overhead after the groups have been given a chance to complete the work.

An example might be as follows:

A candy bar is cut in half and half is given to Nick.



Jacque gets $\frac{4}{5}$ of Nick's half.

If you divide each half into 5 parts, there would be 10 sections formed.



Jacque gets $\frac{4}{10}$ of the whole candy bar. If you rearrange the $\frac{4}{10}$, then the students can see this is the same as $\frac{2}{5}$ of the candy bar.

Allow students to use the grid paper to illustrate and solve the following problems and then create the rule of multiplication for each. Remind students to determine if the product they calculate is a reasonable answer. After students have an opportunity to complete the problem set, randomly select students to share their answers and reasoning. Help students understand that each factor must first be written in fraction form. Next, multiply the two numerators to get the product's numerator. Then, multiply the two denominators to get the product's denominator. Last, simplify as needed.

$$3 \times \frac{5}{6} \quad 2 \times \frac{7}{8} \quad \frac{1}{2} \times \frac{1}{4} \quad \frac{2}{3} \times \frac{1}{2} \quad 3\frac{1}{2} \times \frac{1}{2}$$

Some students may choose to use the commutative property because the problem is easier to model. Make sure students can create real-life situations that will describe each of the problems.

Students should respond to the following in their math *learning log* ([view literacy strategy descriptions](#)):

When you multiply two nonzero whole numbers, the product is equal to or larger than the factors. Is the product of two fractions larger than the fraction factors? Explain your reasoning.

Once students have an understanding of multiplying fractions with visual aids, they need to move to multiplying fractions using a set of rules or an algorithm. Have students work in pairs to explain in words and mathematical symbols how to multiply fractions. Remind them to include instructions that explain how to deal with whole numbers and mixed numbers. After students have time to work, have them share their versions of the rules. Ask probing questions where there are mistakes in student understanding to allow students to discover their mistakes.

Students should be able to describe the following steps:

1. Change any whole numbers to a fraction by writing the whole number as the numerator and 1 as the denominator.
2. Change any mixed numbers to improper fractions by multiplying the denominator by the whole number and adding the product to the numerator to get the numerator of the improper fraction; the denominator will remain the same.
3. Multiply the numerator of the first fraction by the numerator of the second fraction. This is the numerator of the product. Then multiply the denominator of

the first fraction by the denominator of the second fraction. This is the denominator of the product.

4. Simplify, if possible, and change improper fractions to mixed numbers.

The Multiplying Fractions BLM contains additional problems for student practice.

Activity 4: The Meaning of Division of Fractions (GLEs: 5, 8, 40)

Materials List: pencil, paper, Dividing Fractions BLM

In multiplication, most students understand that 4 groups of 2 objects give a total of 8 objects. They need to relate division of fractions to their understanding of the division problem, $8 \div 4$. Students have difficulty in stating the meaning of division -- take a total of 8 candy bars and divide the bars among groups of 4 students, or 8 separated equally into 4 groups, which means that each group of 4 students gets 2 candy bars. Write a problem on the board. Have students write a situation for the problem, and then solve. Repeat the process several times.

Extend student understanding to include division with a fraction: $8 \div \frac{1}{2}$ might mean 8 candy bars divided or separated into half pieces with the answer indicating how many half pieces there would be after the division. Have students predict if the answer will be more than or less than 8, and then let one student model the problem for the class using a picture and words. The picture helps students see division – that 8 candy bars broken in half would result in 16 pieces. Instruct students to return to the predictions they made. Allow several students to share their prediction, and indicate whether it was reasonable or not. Repeat the process using several examples. Record all problems on the board with the intent that one or more of the students will see a pattern which can be written as a rule after doing a series of problems. (Multiply by the reciprocal or multiplicative inverse.) Remind students of the rules they wrote for multiplying fractions. Have them write a rule for dividing fractions. Student should be able to describe the following steps:

1. Change any whole numbers or mixed numbers to fractions.
2. Leave the first fraction alone.
3. Replace the division sign with a multiplication sign.
4. Write the multiplicative inverse or reciprocal of the second fraction.
5. Multiply the two numerators, and then multiply the two denominators.
6. Simplify the quotient as needed.

Take time to discuss students' methods before moving on to dividing fractions by fractions and dividing fractions by mixed numbers. Writing word problems is always difficult, especially with fractions! Just make sure the students attach labels to the fractions so that the problems make sense.

Once students have created this “new” rule for dividing fractions, ask them to demonstrate their understanding of dividing fractions by completing a *RAFT* writing ([view literacy strategy descriptions](#)) assignment. This form of writing gives students the

freedom to project themselves into unique roles and look at content from unique perspectives. From these roles and perspectives, *RAFT* writing is used to explain processes, describe a point of view, envision a potential job or assignment, or solve a problem. It's the kind of writing that when crafted appropriately should be creative *and* informative.

Ask students to work in pairs to write the following *RAFT*:

- R* – Role (role of the writer—Mr./Mrs. Multiplicative Inverse)
- A* – Audience (to whom or what the *RAFT* is being written—6th grade or 7th grade students who do not know how to divide fractions)
- F* – Form (the form the writing will take, as in letter, song, etc.—Job Description or Descriptive Jingle)
- T* – Topic (the subject/focus of the writing—explain the role of the multiplicative inverse when dividing fractions)

When finished, allow time for students to share their *RAFTs* with other pairs or the whole class. Students should listen for accuracy and logic.

The Dividing Fractions BLM contains additional problems for student practice.

Activity 5: Decimal Positioning (GLEs: 5, 8, 9)

Materials List: pencils, chart paper, scissors, glue or tape, learning log

The decimal position of the two factors in a multiplication problem affects the product of two numbers. The following situation will help to build a deeper understanding of this concept.

Give each group of 4 students chart paper, scissors, and glue or tape. Instruct the students to give an example to each of the following (1-4); students should also cut out and paste a model showing each situation on chart/paper.

1. Give an example of a situation that has a product of 56.
2. Give an example of a situation that has a product of 5.6.
3. Give an example of a situation that has a product of 0.56.
4. Give an example of a situation that has a product of 0.056.
5. Explain how answers were derived. Be prepared to present methods used to the class.

As the students present their methods, ask questions to develop the meaning of multiplication of decimals, not just placement of decimal points. For example, students should be able to use the knowledge that 7 groups of 8 pizzas is 56 total pizzas while 0.7 groups of 8 pizzas means a little over half of eight pizzas or 5.6 pizzas. Present more situations like the one above for the students to internalize the rules they will use for multiplication of decimals.

Include a discussion about estimating and reasonable answers. Ask when is it better to use an estimate vs. an exact answer.

Students should respond to the following in their math *learning log* ([view literacy strategy descriptions](#)) without working the problem.

Explain whether the exact product of $(1.4)(0.999)$ will be greater than or less than the estimate $(1.4)(1)$. How can you tell without multiplying 1.4 and 0.999?

Teacher note: Ask questions to make sure the students relate the estimate to the multiplicative identity.

Activity 6: Decimal Division (GLEs: 5, 8)

Materials List: pencils, Decimal Division BLM

Ensure that students develop a conceptual understanding of division of decimals, not just to move the decimal so many places, then divide.

In Activity 4, students wrote situations in order to understand the concept of dividing fractions. The problem $24 \div 6$ can be written as 24 cookies divided among 6 people. How many cookies does each person get, or how many sets of 6 cookies are in a package of 24 cookies? Discuss the meaning of this problem.

While students will know the answers to the problems below, the intent is to develop a conceptual understanding of the placement of the decimal in the answer of a division problem. Give each student a copy of the Decimal Division BLM and have him/her work to come up with the patterns they see in the following problems. Remind students to check for reasonable answers.

1. Nikki has \$25.
 - A. How many 50-cent pieces are in \$25? Write this as a division problem and solve it.
 - B. How many quarters are in \$25? Write this as a division problem and solve it.
 - C. How many dimes are in \$25? Write this as a division problem and solve it.
 - D. How many nickels are in \$25? Write this as a division problem and solve it.
 - E. How many pennies are in \$25? Write this as a division problem and solve it.

Discuss the patterns that students find. Allow students to explain/justify their thought process.

2. Kenneth has \$0.50.
 - A. How many 50-cent pieces are in \$0.50? Write this as a division problem and solve it.
 - B. How many quarters are in \$0.50? Write this as a division problem and solve it.
 - C. How many dimes are in \$0.50? Write this as a division problem and solve it.
 - D. How many nickels are in \$0.50? Write this as a division problem and solve it.
 - E. How many pennies are in \$0.50? Write this as a division problem and solve it.

Discuss the patterns the students find when fifty cents is used, and pose these questions:

How many one dollars are in a quarter?

Does the pattern you found earlier fit this situation? Is it reasonable to have a decimal answer or a whole number answer?

This will cause a bit of concern for the students, because there are no dollars in a quarter; a quarter is a fraction of a dollar. This is where placement of the decimal comes in for division of decimals. More situations like the one above will be needed for the students to get a good understanding of dividing decimals.

Activity 7: Order of Operations—Is It Possible? (GLEs: 3, 5)

Materials List: a number cube or spinner for each student, pencil, Is It Possible? BLM for every student, calculators

Present students with these two sets of problems: $3 + 4 \times 10$; $(3 + 4) \times 10$ and $3 + 2^2$; $(3 + 2)^2$. Have students evaluate each expression. Ask students to compare the two expressions by writing an inequality using $<$, $>$, or $=$. Use student work to lead a class discussion about the use of parentheses and exponents when simplifying expressions. Use a mnemonic such as “Please Excuse My Dear Aunt Sally” to help students remember the order of operations. The first letter of each word corresponds to the first letter in the mathematical operations in the order they are to be performed: Parentheses, Exponents, Multiplication and Division from left to right, then Addition and Subtraction from left to right. Students may have heard other versions from other teachers or created their own version at some point. If students have a difficult time remembering the order of operations, you may want each student to create a mnemonic that is more personal to him/her. Have students work several problems like the two earlier ones before continuing the activity.

Give each student a copy of the Is It Possible? BLM. Instruct students to play Game 1; have students randomly select 4 numbers by either rolling a number cube or spinning a spinner which contains number outcomes. Ask the students to use each of the 4 numbers only once, along with any operations symbols or grouping symbols, to write mathematical expressions that are equal to each of the numbers 1-9. (You may want to allow the students to combine digits to form numbers. Example: I rolled a 3, 6, 2, and 3. I can combine the digits to make $23 - 3 \times 6 = 5$.) Have students check their answers on a calculator. Instruct students to exchange papers to check one another’s work.

When students have completed Game 1, ask students if it will always be possible to write expressions for each counting number using the 4 numbers? (*NO*) Have students share with the class examples of what they believe to be impossibilities for creating each of the numbers 1-9 from four numbers generated by the rolls or spins. Challenge students to see if they can form any of the impossible numbers.

Explain to students that in Game 2 they must use a fraction. Example: I rolled a 3, 6, 2, and 3. I then roll a 5th number to combine with the last 3 to create a fraction. My 5th number was a 3, so I created $(6 - 3 - 2)\left(\frac{3}{3}\right) = 1$. Instruct students to play Game 2 and then exchange papers with another student to check their work. Ask the class to discuss which numbers are impossible to form. Repeat the sequence of events for Game 3 which uses a decimal.

Activity 8: Using Symbols and Multiple Operations (GLE: 3)

Materials List: Challenge Numbers BLM, Challenge Symbols BLM, two boxes, large paper and markers or student white boards and dry erase markers, math learning log

To prepare for this activity, the teacher needs to copy each BLM and cut out the pieces. The pieces from each BLM should be folded and dropped into separate boxes. Include as many copies as needed. Additional numbers may be used.

Review the order of operations. Have students, working in groups of four, select 5 numbers and 4 symbols (operation and grouping) from separate boxes and create an expression that they will use to challenge other students' understanding of the order of operations. Instruct students to write their expressions on large paper or on white boards to present to the class.

To play Challenge, each group will present its expression to the class. The other groups will have three minutes to solve each problem. Have groups write their answers on large paper using markers or small white boards using dry erase markers. When time is called, have each group show its answer. The team presenting will earn one point for each group with an incorrect answer. Be sure to have each group present the problem it created. The group with the most points at the end of the game wins.

Students should respond to the following in their math *learning log* ([view literacy strategy descriptions](#)).

Two students were asked to compute $36 + 12 \div 2 \times 3 - 4$. Sam started by multiplying 2 by 3. Jared started by dividing 12 by 2. Who used the correct procedure? Explain your choice.

Inexpensive 4' x 8' sheets of white tileboard (normally used in bathrooms) are available at home improvement stores and contain the same material used to make more expensive white boards available through school supply stores. Personnel at the store will usually cut the sheets into 2' x 2' squares for a nominal fee.

Activity 9: Problem Solving Triangle Puzzle (GLEs: 5, 7, 10, 11)

Materials List: Triangle Puzzle BLM for each student, scissors, tape, pencil, paper

Provide students with Triangle Puzzle BLM formed of equilateral triangles. Have students cut the triangles apart, and match each problem to the solution. The triangles will form a symmetrical shape when each problem is answered correctly. Students can then tape the pieces together.

Directions for teacher to make additional puzzles: Cut out several equilateral triangles and place together to form a symmetrical shape. Write a problem along one side of a triangle. Find the triangle that shares this side and write the solution along the side of this triangle. Continue this process until all triangles have either an answer or a problem written on each side. Include ratio, proportions, order of operations, percents, decimals, fractions and mixed number situations on the puzzle. Make sure that the same answer is not used more than once as this makes the puzzle very difficult to solve.

Activity 10: Integer Target Part One (GLE: 7)

Materials List: color markers, newsprint or bulletin board paper, yardstick, green markers/counters, Integers BLM, pencil, paper, math *learning logs*

To prepare for this activity, copy the two pages of the Integers BLM on construction paper, cut out the cards, and place them in a baggie. Create number lines (-30 to 30) on the newsprint or bulletin board paper using colored markers and a yardstick. The cards and number lines will be used in Activity 11. This activity should take approximately one 50-minute class period.

Place students in groups of 2, and give each group a sack of four to six cards made using the Integers BLM (some positive and some negative), a number line, and a green marker/counter.

Discuss ways to find the sum of the cards in the stack. Ask, “If we have no cards, what is our total?” Students should respond that the total is zero. Instruct students to place their marker at zero on their number line.

Each group then turns over the first card in their stack. Ask, “What should groups with a positive card do? What about those with a negative card?” Most groups will recognize that those with positive cards should move to the right on the number line, and those with negative cards should move left. Have one student in each group write an equation to describe the group’s reasoning.

Instruct students to leave their first card up and draw the second card. Ask, “How can you move your marker to the place that shows the sum of your two cards?” After some discussion, students should agree to begin where they left off and move the distance and

direction shown by their second card – right for positive and left for negative. Instruct groups to turn over the third card and continue in this manner until the class is comfortable with adding integers.

Now that each group has a small collection of cards face-up in front of it, it is ready to experiment with giving cards away. Each group chooses a positive card to discard and then find the new sum. Have groups write an equation to describe their thinking. If a group holds the cards -2, 4, 3, and -1, and it gives away the 3, it may write “ $-2 + 4 + -1 = 1$,” reflecting the fact that it has discarded the 3 and added the remaining cards. Another group may reason that its original sum of 4 will be reduced by 3 and write “ $4 - 3 = 1$.”

Continue the process, this time discarding a negative card. The students discover that since adding negative cards lowers the total, giving up negative cards must increase the total.

Students should respond to the following prompts in their math *learning logs* ([view literacy strategy descriptions](#)).

1. Describe one or two ways that you can move to the right on the number line by getting or giving away a card. (*Sample answer: I can move to the right on the number line by either getting a positive card or giving up a negative card.*)
2. Describe one or two ways that you can move to the left on the number line by getting or giving away a card. (*Sample answer: I can move to the left by either giving up a positive card or getting a negative card.*)

Once students understand these basic concepts and become familiar with the cards and number line, they are ready to begin playing “Integer Target.”

Integer Target Part One and Integer Target Part Two were adapted with permission from “Integer Target: Using a Game to Model Integer Addition and Subtraction” by Jerry Burkhart, math specialist and teacher, Mankato Area Public Schools, Mankato, MN. *Teaching Mathematics in the Middle School*, March 2007, Volume 12, Issue 7, page 388.

Activity 11: Integer Target Part Two (GLE: 7)

Materials List: Integers BLM and number lines from Activity 10, Integer Target BLM, a red and a green marker/counter for each student, dice

Allow students the maximum amount of play time as the more they play, the more comfortable with the concepts they become.

Students will play the Integer Target game in pairs or groups of four. Each student will need a red and a green marker/counter, a number line (-30 to 30), and a copy of Integer Target BLM. Each group will need a die and a set of cards (one copy of Integers BLM copied on construction paper).

Have students recall their discoveries of how to move to the right and to the left on the number line. If needed, review several “moves” from the previous activity as a group. Explain to the students they will play a game called Integer Target where the moves are similar to the previous activity. Instruct student groups to read the instructions found on Integer Target BLM. Play a mock game as a class, then have student groups play.

Follow Up: The next day, reinforce and extend what has been learned by having students discuss their actions and express them as number sentences. Have students write their number sentences on the board and describe the actions they represent. Be sure to have the students give a solution. (Example: Given the expression $1 - ^{-}4$, students might say that a player begins with a card sum of 1 and gives away a $^{-}4$ card, resulting in a new sum of 5.) Make sure that some examples require students to perform this task in reverse, beginning with the concrete action and finding the equation that would describe it. (I have a sum of 5, and I want a sum of 2. What should I do?) During the discussion, if necessary, lead students to discover that expressions like $4 - ^{-}3$ could be written as $4 + 3$ since both expressions have the same answer. This discovery should lead students to the conclusion that subtraction is the same as “adding the opposite.” Help students understand that although the two expressions are equivalent in the sense that they give the same result, they still have different meanings.

Have the students play the game a second time. Now, during their turn, they must write an equation that describes the action they took. The form of the equation is “beginning total” +/- “action card” = “new total.”

A computer version of *Integer Target* is available at www.integertarget.com.

Activity 12: Integers All Around (GLE: 7)

Materials List: paper, pencil

Provide the students with several different real-life problems involving integers. Example situations might include gaining and losing yards in football, temperature change on a thermometer, number cubes with positive and negative numbers, game shows which involving making or losing money, or a dart game with positive and negative amounts on the board. Have the students work the problems, and check their answers to see if they are reasonable.

Have students work in pairs to create 2 to 3 of their own problems. Instruct groups to exchange and solve the problems. Remind students to check their answers for reasonableness.

Activity 13: Cooperative Problem Solving (GLEs: 5, 7, 10, 11)

Materials List: Cooperative Problem Solving BLM, pencil, paper

To prepare for this activity, copy the Cooperative Problem Solving BLM, and cut the pieces apart. Problems may be separated by placing them in sandwich bags to be distributed to the student groups.

Have students work in groups of two or three to solve real-life situations. Each group should be given one sandwich bag which contains the pieces of one word problem. Each student takes at least one card and keeps it in his/her possession. Have students in each group take turns sharing the information on their cards, then work together to find a solution to the situation. This is a good tool to get all students involved in the problem-solving process. Even the weakest students have a part, because they must contribute the information on their cards and read them to the group in order for the problem to be solved.

Example of a set of cards that one group would solve:

The seventh graders are planning to sell cups of hot chocolate at the basketball games this winter.

If 6 spoonfuls of mix make a cup of hot chocolate,

How many spoonfuls of mix will be needed to make 42 cups of hot chocolate?

Allow time for student groups to share their problem and solution. Lead a class discussion about the different methods used to solve the problems. Students should be able to identify the quantities being compared; point out that when two quantities are compared and written as a fraction, that fraction is called a ratio. Have students find equivalent ratios in the problem they worked. Model how to set up and solve proportions using the problem set in this activity. Remind students that each proportion is two equivalent ratios.

Activity 14: Common Ratios (GLEs: 5, 10, 11)

Materials List: a measuring tape for each group, Common Ratios BLM for each student, pencils, calculators

Remind students how they used proportions in the previous activity to solve problems when equivalent ratios could be written. Include examples to reinforce this concept.

Students will compare their heights with other measurements of their body to the nearest millimeter to determine if there is a common ratio. Be careful with the division of groups during this activity. Give each student a copy of the Common Ratios BLM for recording measurements. Have students work in groups of three or four. Ask them to take turns

with one student measuring, one student recording on the given chart, and the third being the person measured. Have students complete the chart by measuring the distances described in the chart, and then finding each ratio. Have the students compare their findings to the findings of the other students in the group. Then have the students complete the questions as a group.

Have students research the work of Leonardo da Vinci to see if their proportions relate to his ratios. Have students use da Vinci's ratios to predict their measurements. What is The Golden Ratio? How do artists use these ratios today? Websites which provide information on the Golden Ratio include these:

<http://www.geocities.com/jyce3/>,

http://members.tripod.com/mropfer/the_golden_webquest.htm, and

http://en.wikipedia.org/wiki/Golden_ratio.

Activity 15: In Another World! (GLEs: 5, 11)

Materials List: In Another World! BLM for each student, rulers and/or measuring tapes, pencils, calculators

You are a 65-inch tall Earthling who has landed on the world of Gianormas. Immediately upon arrival, you meet Leonardo who is 50 ft tall! As you look around, you notice that everything in this new world is Leonardo's size. You assume that everything is to the same scale as it is on Earth.

Students will measure items that may be found in the classroom to the nearest quarter-inch. Then they will use proportions to find the measurement of the same items in a world of giants, then in a world of miniatures. Each student will need a ruler or measuring tape, a pencil, and a calculator to complete the In Another World! BLM. Remember to convert inches into feet or vice versa when necessary.

Activity 16: Additive and Multiplicative Patterns (GLE: 40)

Materials List: pencil, paper

Give the students the following situations:

- The clock in the classroom loses 5 minutes every day. If this pattern continues, how much time has the clock lost in 2 weeks? Explain the pattern.
- Rachel earned \$2.00 for delivering newspapers the first week of her job, \$4.00 the second week, and \$8.00 the third week. If this pattern continues, how much will she make in 10 weeks? Explain the pattern.

Have the students describe the patterns as additive or multiplicative patterns and provide justification. Have students create similar examples of additive or multiplicative patterns.

Sample Assessments

General Assessments

- Whenever possible, create extensions to an activity by increasing the difficulty or by asking “what if” questions.
- The student will be encouraged to create his/her own questions.
- The student will create and demonstrate math problems by acting them out or using manipulatives to provide solutions on the board or overhead.
- Use the website <http://www.rubrics.com> to create a rubric to assess student work.
- The student will complete journal entries using such prompts as:
 - Explain whether the exact product of $(1.40)(0.099)$ will be greater than or less than the estimate $(1.4)(1)$. How can you tell without multiplying 1.4 and 0.099?
 - Using the numbers -6 , $\frac{1}{2}$, -2 , and 5 and any three operations, what problems can you write for which the answer fall between 2 and 0?
- The student will create a portfolio containing samples of his/her ability to work problems such as the following:
 - While watching the LSU football game, Jerrica became very thirsty. The snack stand sold drinks in 4 sizes.
Kiddie Size: 20 ounces for \$0.80 Adult Size: 32 ounces for \$0.90
Super Size: 44 ounces for \$0.99 Super Duper Size: 64 ounces for 1.25

For which size would Jerrica get the most drink for her money? Explain how you made your decision. If the snack stand offered a Mega Size drink, how much should Jerrica expect to pay? Explain your thinking.

Activity-Specific Assessments

- Activity 3: The student will solve this problem: Monica needed to triple a recipe for cookies. The recipe called for $2\frac{1}{2}$ cups of flour and $1\frac{3}{4}$ cups of sugar. How much will she need of each? Prove your answer. Explain how this problem illustrates multiplication of fractions.
Solution: To triple a recipe means to multiply by 3. Three groups of $2\frac{1}{2}$ for flour, and 3 groups of $1\frac{3}{4}$ for sugar.
- Activity 4: The student will solve this problem correctly: A local coffee house donated twelve pounds of fresh-roasted coffee. The students are running a fundraiser at school and decide to sell the coffee in bags. How many bags can be made if each bag contains $\frac{3}{5}$ pound? $\frac{1}{8}$ pound? Explain how you arrived at your answer.

Solution: Twelve divided into bags that are $\frac{3}{5}$ of a pound. Students can draw a picture of the situation. Twelve divided into bags that are $\frac{1}{8}$ of a pound. Students can draw a picture of the situation.

- **Activity 5:** The student will solve this problem correctly: Carlos placed a bunch of grapes on a scale at a fruit stand. The bunch weighed 2.7 pounds. Grapes are on sale for \$1.59 a pound. Suppose Carlos has a \$5 bill. Does he have enough money to buy the grapes? Justify your answer.

Solution: $2.7 \times \$1.59 = \4.29 Yes, Carlos has enough money.

- **Activity 8:** The student will respond to the following situation either to turn in or to write in their math *learning log* ([view literacy strategy descriptions](#)). Three students were asked to evaluate the following expression. $46 - 30 \div 2 \times 3 + 6$ Which solution do you think is correct?

Meghan's solution
 $46 - 30 \div 2 \times 3 + 6$
 $16 \div 2 \times 3 + 6$
 $8 \times 3 + 6$
 $24 + 6$
 30

Quinton's solution
 $46 - 30 \div 2 \times 3 + 6$
 $46 - 30 \div 6 + 6$
 $46 - 5 + 6$
 $41 + 6$
 47

Lane's solution
 $46 - 30 \div 2 \times 3 + 6$
 $46 - 15 \times 3 + 6$
 $46 - 45 + 6$
 $1 + 6$
 7

Explain why you agree with the solution you selected.

- **Activity 14:** Look for completeness of charts with body measurements and ratios along with a written summary of discovery.
- **Activity 15:** The student will solve this problem correctly: At a local Middle School there are 784 students; of that there are 386 boys and 398 girls. Half of the boys in the school are involved in sports at school. There is a 4:1 ratio of boys to girls who play sports at the school. About how many boys and how many girls play sports at the middle school? Is the ratio of boys to girls who play sports at the school the same ratio as the number of boys to girls in the entire school?

Solution: 49 girls play sports at the school. No, 198 boys to 49 girls play sports at the school; the ratio of boys to girls at the school is 386 to 398.

Grade 7
Mathematics
Unit 3: Patterns, Computation, and Algebra

Time Frame: Approximately four weeks



Unit Description

This unit ties numerical problem solving to algebraic problem solving. Starting with computations using the distributive property, the unit moves into solving and graphing solutions to equations and inequalities, and graphing on a coordinate grid. Function machines will be used to analyze and discover the relationships of changing growths and patterns. Relationships among units and conversions between units within the same system will be addressed.

Student Understandings

It is here that students gain an understanding of exponents of 2 and 3 and the evaluation of expressions containing these exponents. These are tied to square and cube root applications in geometry. Students should also be able to identify and interpret the relationship between squares and cubes and their roots for real number situations involving perfect squares. They should be able to use mental math to match algebraic inequalities with the situations they model, particularly in using inequalities to approximate the values of square and cube roots that are not perfect. They should be able to apply the distributive property of multiplication over addition to solve problems.

Guiding Questions

1. Can students match squaring and cubing numbers with square roots and cube roots (of 8 and 27) and solve both in whole number settings?
2. Can students link algebraic inequalities with their verbal descriptions?
3. Can students approximate the square and cube roots of numbers using inequalities and perfect squares and cubes?
4. Can students evaluate expressions involving exponents of 2 and 3?
5. Can students solve and extend patterns involving exponents?
6. Can students apply the distributive property?
7. Can students graph points on a coordinate grid?

Unit 3 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
3.	Solve order of operation problems involving grouping symbols and multiple Operations. (N-4-M)
4.	Model and apply the distributive property in real-life applications (N-4-M)
Algebra	
12.	Evaluate algebraic expressions containing exponents (especially 2 and 3) and square roots, using substitution (A-1-M)
13.	Determine the square root of perfect squares and mentally approximate other square roots by identifying the two whole numbers between which they fall (A-1-M)
14.	Write a real-life meaning of a simple algebraic equation or inequality, and vice versa (A-1-M) (A-5-M)
15.	Match algebraic inequalities with equivalent verbal statements and vice versa (A-1-M)
16.	Solve one- and two-step equations and inequalities (with one variable) in multiple ways (A-2-M)
17.	Graph solutions sets of one-step equations and inequalities as points, or open and closed rays on a number line (e.g., $x = 5$, $x < 5$, $x \leq 5$, $x > 5$, $x \geq 5$) (A-2-M)
18.	Describe linear, multiplicative, or changing growth relationships (e.g., 1, 3, 6, 10, 15, 21, ...) verbally and algebraically (A-3-M) (A-4-M) (P-1-M)
19.	Use <i>function machines</i> to determine and describe the rule that generates outputs from given inputs (A-4-M) (P-3-M)
Measurement	
22.	Convert between units of area in U.S. and metric units within the same system (M-5-M)
Geometry	
29.	Plot points on a coordinate grid in all 4 quadrants and locate the coordinates of a missing vertex in a parallelogram (G-6-M) (A-5-M)
Patterns, Relations, and Functions	
39.	Analyze and describe simple exponential number patterns (e.g., 3, 9, 27 or 31, 32, 33) (P-1-M)

Sample Activities

Activity 1: Developing the Distributive Property (GLE: 4)

Materials List: pencil, paper

Start this activity by having the students solve one of these problems: 34×8 or 157×5 . Next, ask students to explain their methods for solving the given problem. Students may show methods such as repeated addition or just regular multiplication. Students may use the distributive property and not know that they have done so. If students do not mention the distributive process or describe the use of the property for solving such a problem, ask the class if it would be possible to work 34×8 by finding 30×8 and adding this answer to 4×8 . If they agree that it is possible to do this, have the students try 157×5 using the same method.

One way to efficiently multiply a two-digit number by a one-digit number is to use the distributive property. Provide several problems which require the use of the distributive property (e.g., $6(24) = 6(20 + 4) = 6(20) + 6(4) = 120 + 24 = 144$). Remind students that rewriting 24 as $20 + 4$ is writing the number in expanded form, a process they learned in grade school.

Do not allow students to use pencil, paper or calculators for the next problem. Put the following problem $(6 \times 84) + (6 \times 16)$ on the board allowing only 5 - 10 seconds for student think time. Students will not be able to find a solution in the time allowed because they are trying to multiply 6 and 84, then multiply 6 and 16, and add the two. Give students a hint that the distributive property could be used to work this problem quickly. Let students see if they can come up with how the distributive property could be used. $(6 \times 84) + (6 \times 16) = 6(84 + 16) = 6 \times 100 = 600$.

After practicing several forms of distributive property problems, ask: "Will the distributive property work when dealing with subtraction?" Have the students prove why or why not by making up problems to show proof. (*Yes, it does work:*

$$(7 \times 73) - (7 \times 33) = 7(73 - 33) = 7 \times 40 = 280$$

Will the distributive property work with fractions? Try this one: $\frac{1}{2} \times 6\frac{2}{3}$. Hopefully, this will surface: $(\frac{1}{2} \times 6) + (\frac{1}{2} \times \frac{2}{3}) = 3 + \frac{1}{3} = 3\frac{1}{3}$. We take half of 6 and half of two-thirds, then just group the answers together. It might not always be this easy, but it will work.

Pair students and have them develop their own examples using whole numbers and fractions with addition and subtraction. Ask them to share the examples with another pair of students to see if they can solve the problems using the distributive property.

Activity 2: Distribute It With Candy Bars! (GLE: 4)

Materials List: pencil, Candy Bars BLM for each student

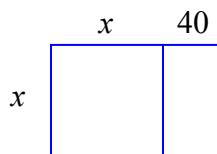
Give each student a copy of Candy Bars BLM. Work through the problems with the students, or have students work in pairs, then have a class discussion about the problems.

The Candy Bar BLM gives the students a scenario that involves selling three types of candy bars. Different types of candy bars are packaged with different numbers of bars in each box. Students must use the distributive property to solve problems about selling the candy bars. Additional problems may be used as needed.

Activity 3: Perimeter of a Corral (GLE: 4)

Materials List: pencil, paper

Have groups of students explore how to determine the perimeter of a rectangular corral that has a width of x feet and a length of $x + 40$ feet.



Have students write the expressions for finding the perimeter different ways, explaining the method each time. Possible examples of expressions include adding all the side lengths, $x + x + (x + 40) + (x + 40)$, showing that there are two lengths and two widths, $2(x) + 2(x + 40)$, and using the distributive property, $2[x + (x + 40)]$. Have a class discussion of the various methods found. Give each group a different value for x and have each student in the group use a different expression to find the perimeter. Ask group members to compare their answers with one another. Did all the expressions give the same perimeter? Why or why not? Repeat this activity using various widths and lengths.

Present the class with a *SPAWN* ([view literacy strategy descriptions](#)) prompt. *SPAWN* is an acronym for six categories of writing prompts to develop student thinking and get them to articulate in their own words what they've learned. Using the W or "What If?" category of *SPAWN*, present students with the following prompt:

Matt's dog needs a fenced yard. Matt's uncle Thomas has 132 feet of chain fence he will give Matt. If Matt uses the back of his house as one side, he only has to put fence around three other sides. His house is 32 feet long. Describe two ways Matt could make a fenced in yard for his dog. Include sketches and dimensions in your description.

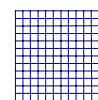
Once students have completed their responses to this *SPAWN* prompt, have them share with a partner or the whole class to stimulate discussion about approaches to solving the problem.

Activity 4: Building a Cube (GLEs: 22, 39)

Materials List: ruler, pencil, scissors, and a brown grocery bag or piece of newsprint for each student, tape, string or yarn, cube-shaped box, solid wooden or plastic blocks, math learning log

This activity lays a conceptual foundation for understanding squares and cubes and conversions within the same system. These two concepts are very difficult for students to grasp.

Give a ruler and a brown paper bag (available at grocery stores) to each student. Have each student draw a 12 inch by 12 inch grid on the paper bag, and then cut out the grid in order to make a square foot. Be sure to also identify the area of the square as $12 \text{ in} \times 12 \text{ in} = (12 \text{ in})^2 = 144 \text{ in}^2$.

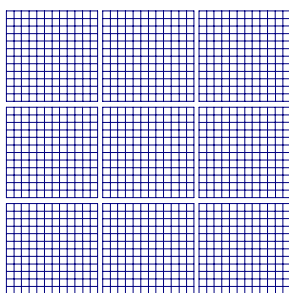


Discuss the conversion. Make sure that students understand that they are changing two of the dimensions, not just one, when converting square inches to square feet and that 144 square inches = 1 square foot.

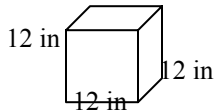
Discuss how students could make a square yard from the square foot grids. A square yard can be made by taping 9 student square foot grids together (three square foot grids by three square foot grids).

Discuss the conversion. Make sure students understand that they are changing two of the dimensions, not just one, when converting square feet to square yards.

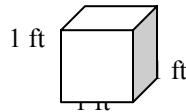
9 square feet = 1 square yard



Tape 6 of the students' square grids together that are 1 foot by 1 foot by 1 foot to form a cubic foot. Explain that the six faces represent only the surface area of the cubic foot; a cubic foot is a solid. To help students visualize this concept, have them fill a cube-shaped box with solid wooden or plastic blocks to allow students to find the volume of the cube they create.



or



$$12in \times 12in \times 12in = (12in)^3 = 1728in^3$$

$$1ft \times 1ft \times 1ft = (1ft)^3 = 1ft^3$$

Discuss with the students how to build a cubic yard. How many cubic feet will be needed to make a cubic yard? More discussion will be needed here for student understanding.

Have students tape 6 of the 3 foot by 3 foot (1 square yard) sections to form a cubic yard. Again, remind students the six faces only represent the surface area of the cubic yard. Discuss the conversion. Make sure students understand that they are changing three of the dimensions, not just one or two, when converting cubic measurements.

If possible, hang the square foot and cubic foot (square yard and cubic yard also) from the ceiling for reference during the year.

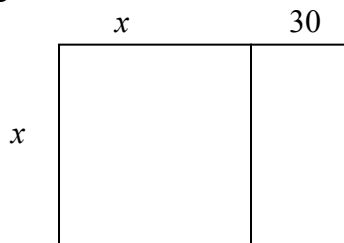
After completing this activity, discuss and analyze the exponential number pattern that is formed by squaring numbers, and cubing simple numbers such as 2 and 3.

In their math *learning log* ([view literacy strategy descriptions](#)), have the students predict the number of cubic inches (in^3) in a cube measuring 2 ft by 2 ft by 2 ft and in a cube measuring 4ft by 4ft by 4ft. Have the students explain their reasoning.

Activity 5: Area of a Corral (GLEs: 4, 12)

Materials List: paper, pencil

Have students work in groups of 2 or 3 to explore how to determine the area of a rectangular corral that has a width of x feet and a length of $x + 30$ feet.



Ask each group of students to choose and substitute a four digit number for x and find the area of the corral. Instruct students to show more than one method for finding the area. (i.e., finding the areas of the two smaller rectangles and then adding them or finding the area of the large rectangle in one step). Make sure students do not leave out the units. Remind students that area is measured in square units: if $x = 400ft$ then,

$$(400ft)^2 + (400ft \times 30ft) = 160,000ft^2 + 12,000ft^2 = 172,000ft^2$$

Once students have completed the problem above, ask them to demonstrate their understanding of area by completing a *RAFT* writing ([view literacy strategy descriptions](#)) assignment. This form of writing gives students the freedom to project themselves into unique roles and look at content from unique perspectives. From these roles and perspectives, *RAFT* writing has been used to explain processes, describe a point of view, envision a potential job or assignment, or solve a problem. It's the kind of writing that when crafted appropriately should be creative *and* informative.

Ask student groups to write the following *RAFT*:

R – Role of the writer—Store Clerk at Sam's Feed and Seed

A – Audience or to whom or what the *RAFT* is being written—Mr. Mobley, owner of corral

F – Form, the form the writing will take, as in letter, song, etc.—estimate

T – Topic or the subject focus of the writing—On company letterhead, create an estimate for Mr. Mobley, the owner of the corral. Include a drawing of his corral, the area of the corral, and the projected cost to plant grass seed in his corral. A 5 pound bag of grass seed will cover 2,000 square feet and costs \$15.00.

Allow time for student groups to share their *RAFTs* with the class and discuss their estimate/work with the class. Students should listen for accuracy and logic in their classmates' *RAFTs*.

Activity 6: Number Line Square Roots (GLE: 13)

Materials List: square tiles, Square Roots BLM or index cards labeled with similar problems, learning logs

Give each student 4 square tiles, and instruct them to use the tiles to make a larger square. Have a class discussion in which students explain what they know about the new square. (*Sample Answer: The new square is 2 tiles by 2 tiles. Its area is 4 square tiles.*) Discuss the concepts of perfect squares and square roots. Squaring a number and finding the square root are inverse operations just as addition and subtraction “undo” each other. Indicate that a perfect square number gets its name because you can make a square with that number of given tiles; therefore, 4 is called a perfect square. You cannot make a square with 5 tiles, so 5 is not a perfect square. Another way of knowing if a number is a perfect square is by determining its square root. The square root of a perfect square is a whole number. $\sqrt{4}=2$; $2^2=4$. Be sure students can distinguish between squares and square roots.

Now, have two students use their combined blocks to make the biggest square possible. Have groups explain their solution and reasoning. The largest square possible measures 2 by 2; the next biggest square would be 3 by 3 or 9 square tiles. The students only have 8 tiles. 8 is not a perfect square; there is no whole number that can be multiplied by itself (squared) to get 8.

Have students use other sums of square tiles to model perfect squares. Instruct students to find the first twenty perfect squares; $1^2=1$, $2^2=4$, $3^2=9$, $4^2=16$, and so on.

Draw a number line on the board and number it from -20 to 20 (or more) on which students will determine the placement of square roots. Cut apart the problems on the Square Roots BLM or put similar problems on index cards. Examples of problems: What is the approximate value of $\sqrt{6}$? What is the value of $\sqrt{64}$? Make sure to have several types of problems.

Pass out all cards to the students. Have a student read his/her card aloud, and place a mark on the number line where he/she thinks the answer belongs.

Have a class discussion about the placement of the number on the number line each time. Ask questions regarding the placement of the answers. Example: Is your answer for the $\sqrt{6}$ closer to 2 or 3? Why? Continue until all problems have been read aloud and the answers placed on the number line.

Have students respond to the following prompt in their math *learning logs* ([view literacy strategy descriptions](#)):

Is $\sqrt{21}$ closer to 4 or 5? Justify your reasoning with words, numbers, and a number line.

Activity 7: Four in a Row (GLEs: 3, 12, 13, 29)

Materials List: overhead, overhead pens, coordinate grid transparency, pencils, graph paper, cup

Show students how to simplify order of operations problems that contain square roots such as $5x^2 + \sqrt{16} - 2$.

Divide the class into two teams. To start play, put an algebraic expression on the overhead and display a coordinate grid on the board. Have one team give coordinates for a point they wish to capture on the coordinate grid. Circle the point on the coordinate grid. Instruct the team to substitute the coordinates of the point chosen into the algebraic expression. If the team provides the correct answer, award the team that point and fill in the circle with the color chosen for that team. If the team in play cannot provide a correct answer, give the opposing team an opportunity to fill in the circle. For example, give the expression $x + y^2$. Team 1 wants to capture (2, 3) so they substitute the coordinates for the variables in the expression: $2 + 3^2$ and evaluate to get 11. Have teams alternate playing until one team has captured four grid points in a row horizontally, vertically, or diagonally.

Be sure to have several expressions already made up before play begins, and see that the expressions contain exponents (especially 2 and 3) and square roots. Example of expressions: $2x + 3y^2$, $\sqrt{16}x - \frac{y}{2}$, $(x + y)$, $x^2 + 2y$, $3x - y^3$, $2x^2 + \sqrt{25}$

Extension:

After game has been played as a whole class, divide each team into two new teams to play. Before play begins, have each team create 8-10 expressions, cut them apart, and place them in a cup. Provide additional expressions to each game to ensure that all of the skills are being covered. To start play, one team picks an expression out of the cup, chooses the coordinates for the point they wish to capture, then substitutes the coordinates of the point into the expression and evaluates it. The other team must check their work. If a team provides a wrong answer, the other team has a chance to capture the point by providing a correct solution. Teams take turns until one team has captured four points in a row horizontally, vertically, or diagonally.

Activity 8: What's My Value? (GLE: 12)

Materials List: What's My Value? BLM, pencils

Before beginning this activity, lead a class discussion on the building of the cubes in Activity 4 in this unit.

The What's My Value? BLM provides a list of algebraic equations involving exponents of 2 and 3 and square roots. Students are instructed to determine which replacement values from the second column should be used to make the equation in the first column true.

Examples: Equations	Replacement Values
$\sqrt{x} + 5 = 7$	$x = 7$
$x^2 - \sqrt{9} = 46$	$x = 4$
$x^3 + \sqrt{4} = 29$	$x = 3$

After students have completed What's My Value? BLM, lead a discussion about how students found their answers. Most students will use the guess and check method for most of the problems. Suggest the work backwards strategy if none of the students mentions it. Model the strategy using several of the equations from this activity. Then give students some additional equations similar to these to work using the work backwards strategy.

Activity 9: Situations with Equations (GLEs: 14, 16)

Materials List: Equations BLM, newsprint or large paper (2-3 sheets per group of four), markers

Lead a class discussion about algebraic equations. Include examples like those found on Equations BLM. Ask students how they would go about solving one of these. Lead students to solve the equations using symbolic steps.

Have students work in groups of four to make a team. Give each team 2-3 equations from the Equations BLM. Have each team work together to solve their equations on newsprint or large paper, using symbolic steps and/or diagrams to justify their work. Ask students to write a situation that would represent the equations given to them.

Inform the students after the teams have completed their work, they will assume the role of *professor know-it-all* ([view literacy strategy descriptions](#)) and present their work and situations with the class. One member of each team should explain the addition/subtraction grouping, one the multiplication/division grouping, one the checking step, and another the situation that represents the equation. Remind students that as *professor know-it-all*, they must be prepared to defend/prove their reasoning for their work.

When students present, be sure they explain their procedures verbally and that they don't just give written steps. Also, prepare the other students to formulate questions to ask of the know-it-alls and to hold them accountable for the accuracy of their answers.

Instead of using newsprint or large paper, you may want to give each group blank transparencies and let them use the overhead when presenting.

Add equations to this problem set, if needed.

Activity 10: Inequality Bingo (GLE: 15)

Materials List: Inequality Bingo BLM, pens, Verbal Inequalities BLM (cut apart), a bowl or box, Algebraic Inequalities BLM (cut apart), small squares of paper

Version 1. Have students match algebraic inequalities with verbal statements by using a modified *Bingo*[®] game played with a 3 x 3 grid on each card.

Give each student a copy of Inequality Bingo BLM. Point out there are two games on one sheet. On the first *Bingo*[®] card, have students fill two spaces with the symbol $<$, two spaces with the symbol $>$, two spaces with the symbol \leq , two spaces with the symbol \geq and one = symbol in the center (free space).

Randomly select numbered, simple word statements from a bowl or box and read them to the class. Have students mark off the symbol that should be used to represent the relationship indicated in the problem. Also, instruct students to mark the number of the problem on each square as they are filled as this will facilitate the verification of a winner. For example, the teacher reads problem #3 – I have less than Mary. The student is expected to write #3 in one of the boxes that contains a $<$ symbol then cover the symbol. When a student wins the game, let the class verify the winning card. The numbered simple word statements are on the Verbal Inequalities BLM.

Allow students to make up their own word statements that involve greater than, less than, and other comparisons for future games.

Version 2. Extend the activity by having students match algebraic inequalities with verbal statements and vice versa. Give students the list of inequalities found on Algebraic Inequalities BLM, and have them write one in each of the boxes on the second *Bingo*[®] card. Randomly select word statements that are on the Algebraic Inequalities BLM from a bowl, and read them to the class. Have students use a square of paper to cover the inequality that represents the relationship indicated. For example, *if Joe's age, x , added to twice Morgan's age, y , is greater than 26* is read, cover or mark the space containing $x + 2y > 26$ on his/her *Bingo*[®] card. When a student wins the game, let the class verify the winning card.

It would be a good idea to display the sentences on the overhead or where students can see them.

For variation, continue playing any of the versions of the game until a black out is achieved.

Activity 11: Who Has? (GLEs: 14, 15, 16, 17)

Materials List: index cards, transparencies cut into strips

Using index cards, make a set of cards showing verbal statements for one-step equations and inequalities – at least one for each student in the class. Example: My mystery number is greater than or equal to 6. Who has the graph and inequality for my mystery number?

Write answers on transparency strips with one strip having the inequality/equation and another strip showing the graph (as points, or open or closed rays on a number line).

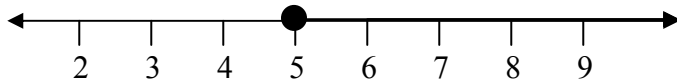
Distribute at least one index card and two transparency strips (a graph and an inequality/equation) to each student. Have one student read the verbal statement aloud. Ask the students with the correct inequality and the graph to place them on the overhead. Continue the process until all cards have been used.

Example

Verbal Statement on index card: My mom says that I have to save at least \$5.00 each time I get paid; who has the graph and inequality for this statement?

Transparency: I have the inequality: $x \geq \$5.00$ (Student puts the inequality on the overhead).

Transparency: I have the graph. (Student puts the graph on the overhead)

**Activity 12: Patterns to Investigate (GLEs: 18,19, 39)**

Materials List: pencil, paper

In this activity, have students describe patterns by making charts, tables or drawing the patterns using a function machine. Ask them to write an expression describing the rule for the numbers in the sequence, and provide the 100th number in the sequence using the rule described.

Start by making a chart or table pairing the term number with each number in the sequence. Indicate that the students are to think of this as a function machine. The object is to try to figure out the rule that the function machine uses.

Table 1

Term number	1	2	3	4	5	6	10	50	100	n
Number in the sequence	6	7	8	9	10	11	15	55	105	? $n + 5$

Have students make observations and discuss what they notice about the table. Ask them to describe and write the rule that generates the output from the given inputs.

In the table below, fill in the missing values for the following function machine.

Table 2

Term Number	1	2	3	4	5	6	100	n
# in the sequence	3 3^1	9 3^2	27 3^3	81 3^4	243 3^5	729 3^6	3^{100}	3^n

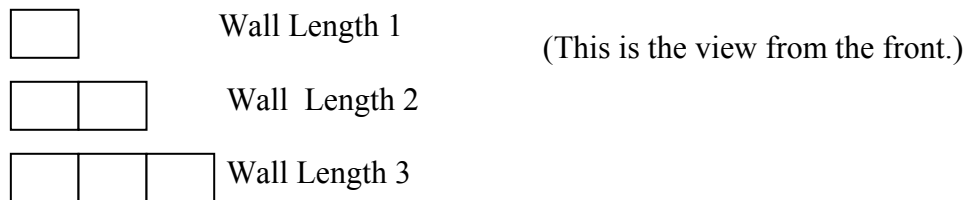
Have students make observations and discuss what they notice about the table. Ask how is it different from Table 1 and to describe and write the rule that generates the output from the given inputs.

Activity 13: More Patterns! (GLEs: 18, 19, 39)

Materials List: toothpicks, pencil, paper

Present the following problem to the students. Have the students work in groups to complete the activity using a function machine.

An engineer designs the skeleton for the walls of a new stadium from equal lengths of steel beams placed in a rectangular pattern shown below. How many steel beams are needed to construct a wall whose length is 57 beams long?



The length of the wall is measured by the number of beams along the bottom of the wall.

- Ask students to model each wall length with toothpicks to represent the beams to a wall length of 6.
- Then have students make a table, record the total of beams for each wall length, and share their observations with their group members.
- Instruct students to predict how many beams are needed for walls of lengths 7 and 10.

Example of table with solutions.

Wall Length	1	2	3	4	5	6	7
Total Number of beams	4	7	10	13	16	19	22

- Have students write sentences to describe the patterns they see in the table (i.e., write a mathematical statement or rule to describe the rule that relates the length of a wall to the total number of beams). Have each group write its rule on chart paper to share with the class.

Some example or rules the student may see: a) The total number of beams is equal to 3 times the number of lengths plus 1. b) The total number of beams is equal to 1 less than the length, times 3, plus 4. c) The total number of beams is equal to 2 times the number of lengths, plus the number of lengths, plus 1.

- e. Have the students describe their rules symbolically using l to represent the length of the wall and b to represent the total number of beams needed to construct the wall.

Example equations that go with the above rules: a) $b = 3w + 1$ b)
 $b = 4 + 3(w - 1)$ c) $b = 2w + w + 1$

- f. Have the students determine how many beams are needed for a length of 57 and a length of n .

Sample Assessments

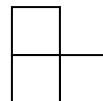
General Assessments

- Determine student understanding as the student engages in the various activities.
- Whenever possible, create extensions to an activity by increasing the difficulty or by asking “what if” questions.
- The student will be encouraged to create his/her own questions.
- The student will create and demonstrate math problems by acting them out or by using manipulatives to provide solutions on the board or overhead.
- Use the website <http://www.rubrics.com> to create a rubric to assess student work.

The student will complete math *learning log* ([view literacy strategy descriptions](#)) entries using such prompts as these:

- Compare and contrast x^2 , $2x$, and $x + 2$.
- Explain the difference between inequality, expression and equation.
- Explain which represents the larger quantity: 2^3 or 3^2 ?
- The student will create a portfolio containing samples to show understanding of concepts learned in the unit. Example:

- Suppose you are developing a pattern. This is the first arrangement in the pattern. When you add squares to the pattern to make it grow, this is the second arrangement in the pattern.



There are several different ways to continue the pattern. Draw what you think the next three arrangements would look like. Make a table of values comparing the arrangement number to the total number of tiles used. Using the table, predict the total number of tiles in the 10th arrangement and in the 100th arrangement.

Activity-Specific Assessments

- Activity 2: The student will respond correctly to the following situation:
The seats in a theater are divided into two sections. Section A has 15 rows of seats. Section B has 13 rows. There are 10 seats in each row. How many seats are in the theater?

Jamal solved the problem using this method. He multiplied to find the seats in each section, and then added to find the total.

Section A $10 \times 15 = 150$

Section B $10 \times 13 = 130$

Total Seats $(10 \times 15) + (10 \times 13) = 150 + 130 = 280$

Rosa thinks her method is correct. She added to find the total number of rows, and then multiplied by the number of seats per row.

Rows in A and B $15 + 13$

Total seats $10(15 + 13) = 10 \times 28 = 280$

Who is correct? Justify your answer.

- Activity 5: The student will work the following problems correctly:
 1. You are building a square pen for Fido the dog. If you have 64 feet of fence, what will the length of each side of your square pen be? Draw a model of this situation.
 2. Your family plans to build a square patio. The plans say that the patio will have an area of 28 square feet. What is the approximate length of each side of the patio? Explain your reasoning and draw a model.

- Activity 11: The student will graph and write a verbal statement to represent each inequality:
 - a) $n > \$18$
 - b) $c < 20$ gallons
 - c) $x \leq 5$ books

- Activity 13: The student will work the following problem correctly:
A pattern of squares is shown below.
 - a. Sketch the fourth and fifth figure in this pattern.
 - b. Create an input-output table comparing the figure number to the number of squares.
 - c. How many squares would be in the 100th figure?
 - d. Write an expression for the number of squares in the n^{th} figure.

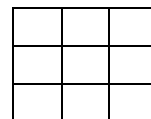
figure 1



figure 2



figure 3



Solutions:

<i>Figure #</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>100</i>	<i>n</i>
<i># of squares</i>	<i>1</i>	<i>4</i>	<i>9</i>	<i>16</i>	<i>25</i>	<i>100²</i>	<i>n²</i>

Grade 7
Mathematics
Unit 4: Surveys, Statistics, and Patterns

Time Frame: Approximately four weeks



Unit Description

This unit focuses on the analysis and interpretation of data. Reasoning with numerical or logic problems will be expanded to include three-circle Venn diagrams.

Student Understandings

Students can distinguish between discrete and continuous data, make appropriate choices in displaying data, and understand the impact this has on interpretations of the graphs. Further, they are able to identify and discuss the significance of gaps, clusters, and outliers as they examine information from surveys and experiments. Students can reason both logically and numerically in situations represented by three-circle Venn diagrams. They use statistical data to reason about the relationship between multiplying the length of the sides of a polygon and its impact on perimeter and area. Students can determine probability from experiments and from data displayed in tables and graphs, and then compare theoretical and experimental probability in real-life situations.

Guiding Questions

1. Can students distinguish between discrete and continuous data and choose appropriate graphical representations?
2. Can students talk about clusters, gaps, and outliers in data and their meanings?
3. Can students analyze and interpret the information in circle graphs?
4. Can students use three-circle Venn diagrams to solving numerical or logic problems?
5. Can students indicate and show how multiplicative changes of side lengths affect the perimeter and areas of polygon?
6. Can the students apply the fundamental counting principle in real-life situations?
7. Can the students determine probability from experiments and from data displayed in tables and graphs?
8. Can the students compare theoretical and experimental probability in real-life situations?

Unit 4 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Data Analysis, Probability, and Discrete Math	
31.	Analyze and interpret circle graphs, and determine when a circle graph is the most appropriate type of graph to use (D-2-M)
32.	Describe data in terms of patterns, clustered data, gaps, and outliers (D-2-M)
33.	Analyze discrete and continuous data in real-life applications (D-2-M) (D-6-M)
34.	Create and use Venn diagrams with three overlapping categories to solve counting logic problems (D-3-M)
Data Analysis, Probability, and Discrete Math	
36.	Apply the fundamental counting principle in real-life situations (D-4-M)
37.	Determine probability from experiments and from data displayed in tables and graphs (D-5-M)
38.	Compare theoretical and experimental probability in real-life situations (D-5-M)
Patterns, Relations, and Functions	
41.	Illustrate patterns of change in length(s) of sides and corresponding changes in areas of polygons (P-3-M)

Sample Activities**Activity 1: Describe the Data (GLEs: 31, 32)**

Materials List: pencil, paper, Describe the Data BLM

Conduct a class survey by asking, “What is your shoe size?” Record the data on the board, but do not organize it. In groups of 2 or 3, instruct students to organize the collected data and to write 3 to 4 sentences to describe/analyze the data. Encourage the students to organize the data graphically to assist them in doing this.

After 10 minutes, have groups present their organization and analysis to the class. Allow the class to check for accuracy and discuss the pros and cons of each representation and analysis of the data. Be sure to include a discussion about patterns, clusters, gaps, and outliers in the data. To help students make connections to the vocabulary, describe examples where they use the same vocabulary every day. They might see a cluster of grapes in the grocery store or a cluster of flowers in their yards. Ask students to describe what a cluster of grapes or a cluster of flowers looks like. Use this knowledge to describe the data in a similar way.

Examples: A cluster of grapes is a bunch of grapes all connected to one big stem. A cluster of data is similar in that there are several pieces of data surrounding a central spot or piece of data. To demonstrate gaps, have students sit in a row using every chair, then

skip several chairs, and then have a student sit in the next chair on the same row. This person is an outlier, and the empty seats represent a gap.

In previous years, students have described data by identifying the patterns they see in the data. Be sure they are able to identify the same patterns using the vocabulary: cluster, gaps, and outliers.

After groups have presented, address the types of graphic organizers not used. Ask why each was not used. Could the information collected be organized using every kind of graphic? Is one better to use than another? Discuss the cons and pros of a line plot, frequency table, bar graph, chart, and circle graph.

Give each student a copy of the Describe the Data BLM. Students will describe/analyze the data in terms of patterns, clusters, gaps, and outliers. After students have completed the Describe the Data BLM, allow them to pair with another student to share their analysis, paying close attention to accuracy.

Activity 2: Circle Graphs All Around (GLEs: 31, 32)

Materials List: examples of circle graphs from magazines and/or newspapers, pencil, paper

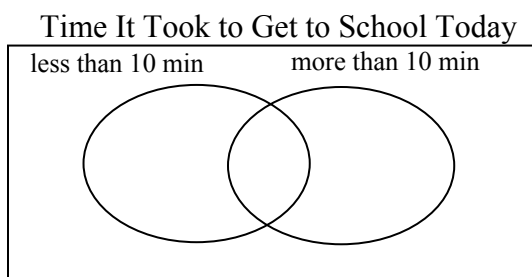
Find circle graphs from magazines and newspapers, and give each group of 4 students a different circle graph to study and interpret. Have students write at least 5 mathematical statements about their graph. After 10 minutes, have each group present their interpretation of the circle graph to the class. When all presentations have been presented, facilitate a discussion of the differences and similarities of the circle graphs presented.

Discuss how to interpret a circle graph and determine when it is appropriate to make circle graphs. Example discussion questions: Does this type of graph lend itself to presenting clusters, outliers, or gaps? When and why are circle graphs used?

Activity 3: Circle Graph: Last Night's Supper (GLE: 31)

Materials List: newsprint (optional), sticky notes (one for each student), pencils, 1-inch wide paper strips, grid paper, scissors, tape, markers (optional)

Have students use a Venn diagram *graphic organizer* ([view literacy strategy descriptions](#)) to organize data. Draw a Venn diagram with two overlapping circles on the board or large piece of paper/newsprint. Label the diagram "Time It Took to Get to School Today." Label one circle "less than 10 minutes" and the other "more than ten minutes."



Give each student a sticky note. Have each student put his/her name or initials on the sticky note. Instruct each student to put his/her note on the Venn diagram based on how long it took him/her to get to school. When all students have placed their notes, discuss what each section represents. Remember the middle represents less than 10 minutes and more than 10 minutes which should be empty. If it takes a student 10 minutes to get to school, the note should go outside of both circles.

Have groups of students work together to create a bar graph of the same information using the strips of paper, grid paper, scissors, tape and markers. Do not allow them to use additional items such as rulers or measuring tapes.

After 10 minutes, have groups present their graphs to the class. Discuss similarities and differences in the graphs. Does this type of graph lend itself to presenting clusters, outliers, or gaps?

Now instruct the students to use only the items they have to create a circle graph. (The goal is for the students to see they can tape the strips of paper end to end to create a circle, then trace around the outside of the paper strips to create a circle.) Ask students to make a mark for the center point of the circle and draw a line from the center point to each point where two strips meet on the edge of the circle. Have the students write a ratio and percent for each section. After about 20 minutes, have students present their graphs to the class.

Discuss how to interpret a circle graph, and determine when it is appropriate to make circle graphs. Example discussion questions: Does this type of graph lend itself to presenting clusters, outliers, or gaps? When and why are circle graphs used?

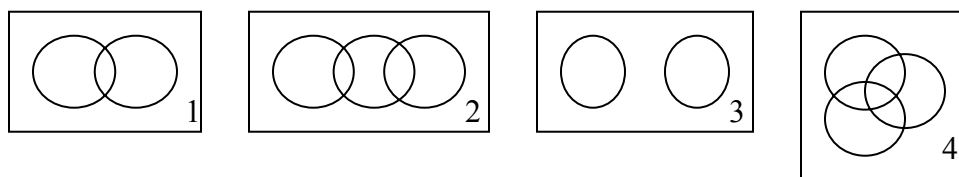
Sentence strips like the ones used in first grade make a great alternative for the 1-inch paper strips when paired with extra-large grid paper. The larger size makes it easier to manipulate the “bars.”

Activity 4: Venn Diagrams (GLE: 34)

Materials List: Venn Diagram Examples BLM, sticky notes (4-6 for each student), pencils

Students have already been introduced to the Venn diagram *graphic organizer* ([view literacy strategy descriptions](#)) in earlier grades. In the 6th grade, students created Venn diagrams with two over-lapping circles. This activity expands this skill to the use of three overlapping circles.

Examples of Venn diagrams are provided below. Copy the Venn Diagram Examples BLMs or create larger versions and place them around the room.



Lead a discussion of the meaning of each diagram.

The first Venn diagram shows two categories that overlap such as *I am wearing blue jeans*, and *I ate breakfast this morning*. Those students who meet both criteria would be listed in the overlapping section. Students who are wearing red and didn't eat breakfast would lie in the inside the rectangle, but not in either circle.

The second Venn diagram shows three categories which overlap: *I ride a bus to school*, *I take more than 10 minutes to get to school*, and *I walk to school*. Have students tell where a student would be placed if they walk to school in 8 minutes or ride 12 minutes in a car.

In Venn diagram three, two categories that have no connection are shown such as the statements: *I am at least 70 inches tall* and *I am less than 64 inches tall*. Ask where to locate a student who is 68 inches tall, 56 inches tall, or 70 inches tall.

The fourth Venn diagram could be used to show the numbers of dogs, cats and fish that students have as pets. Ask why each circle would overlap each of the other circles.

With the class, properly label each Venn diagram with the examples found above or your own. Pass out 4 sticky notes to each student. Each student should place his/her name on each note and then place one on each of the four diagrams.

Discuss the placement of the notes to check for understanding. Lead a discussion regarding trends noticed in the Venn diagrams. Example of the questions that may be asked on the fourth Venn diagram include these: How many people own a dog? How many people own only a dog? What is the most or least popular pet? Have students create their own surveys (e.g., favorite type of movie, favorite type of music), and make a Venn diagram to show the results.

Activity 5: Logic Problems with Venn Diagrams (GLE: 34)

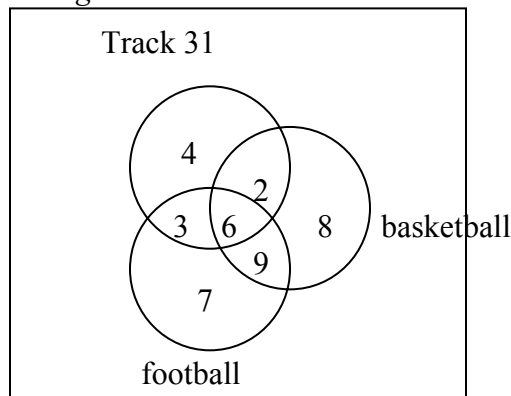
Materials List: transparency, paper, pencil

Have students use logic to create a Venn diagram *graphic organizer* ([view literacy strategy descriptions](#)). Make this a whole class activity and create the Venn diagram on an overhead transparency. Later pair the students to complete other problems that you provide.

- There are 70 students in the 7th grade at Sleepy Hollow Middle School.
- 15 students participate in track
- 25 students participate in football
- 25 students participate in basketball
- 15 students participate in football and basketball
- 9 students participate in track and football
- 8 students participate in track and basketball
- 6 students participate in all three sports

Ask, What size Venn diagram do we need? (*3 overlapping circles*) How do you know? (*There are three activities listed.*) Give students a hint to start with the last statement. Have students discuss and debate where they think the numbers should go based on the information provided. Ask, “How many students do not participate in any of these sports? How do you know?” ($70 - 39 = 31$)

Venn diagram solution



Activity 6: Fast Food Data (GLE: 32)

Materials List: menu with nutritional information from a fast food restaurant (may be found on the Internet), computer with Internet access (optional), grid paper, pencils, dry spaghetti

Students will create a scatter plot of data to explore the relationship between grams of fat and grams of protein in menu items at various fast food restaurants. Allow students to access the Internet to collect their own data or print information to give to students. If

Internet access is not available, most local fast food restaurants have nutritional brochures than can be copied for student use.

<http://www.ntwrks.com/~mikey/chart1.html> or <http://www.calorie-charts.net/> gives nutrition information about specific food items available at the grocery store. http://www.mcdonalds.com/app_controller.nutrition.index1.html gives nutrition facts on menu items at *McDonalds*[®]. <http://www.wendys.com/food/NutritionLanding.jsp> gives nutrition facts on menu items found at *Wendy's*[®]. Choose each menu item individually or click Nutrition Guide in the middle of the page to get facts about each menu item. <http://www.bk.com/#menu=3,-1,-1> is a link to *Burger King*[®]. Click *Nutritional Brochure* to download *Burger King's*[®] nutritional information.

Let students pick at least 5 items they want to chart on a scatter plot. As a class, create a scatter plot showing the grams of fat compared to the grams of protein in individual servings of lunch and dinner items sold at various fast food restaurants. Have the students work with a partner to interpret the data.

Lead a discussion about the patterns in the data. Are there any outliers or gaps? Where does the data seem to cluster? Is there a trend line? Place a piece of uncooked spaghetti on the graph to develop the concept of trend line.

Have students work in pairs to choose 5 additional items from a menu, collect their own data, create a scatter plot, and prepare to discuss any patterns observed in the data. After 15-20 minutes, have student groups take turns presenting their graphs assuming the role of *professor-know-it-all* ([view literacy strategy descriptions](#)). Remind them to write down 2-3 questions they think they will be asked by their peers and 2-3 questions to ask their peers. They also will have to justify their thinking and be prepared to answer questions from the class. Students should hold the know-it-alls accountable for the accuracy of their answers to the questions.

Activity 7: Discrete and Continuous Data (GLE: 33)

Materials List: graph paper, pencils, (beakers, water, thermometers – optional), construction paper

Find examples of continuous and discrete data, and ask what kind of graph, continuous or discrete, would represent each of situations. Examples of data: the number of female bus drivers at each school, the ages of customers for each hour at Penney's, the heights of students in a class, the time it takes an athlete to run 400 meters, and so on.

Discuss and graph the situations with the class. Have students count or provide students with the number of vehicles in the school parking lot along with the make and color of each vehicle in chart format. Discuss the fact that this is an example of *discrete* data since the number of cars can be counted. Lead a discussion about discrete data, emphasizing the inability to have a portion of a car or truck. Discrete data are distinct values. Discrete

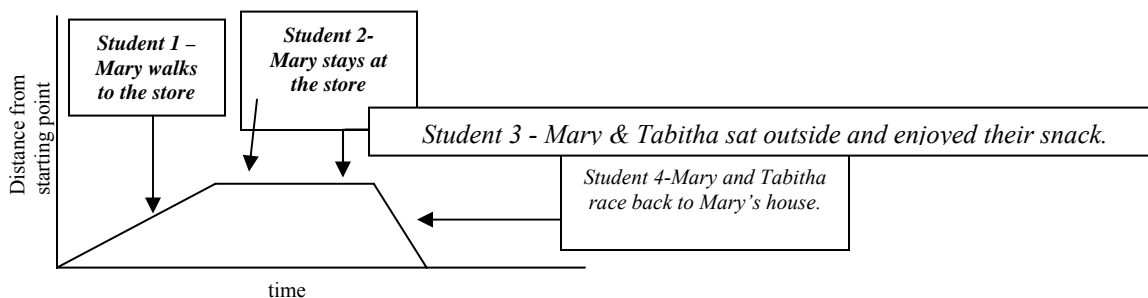
data can be counted, but they do not have to be whole numbers. You can tell if the data is discrete if you can count the number of possible values or if there is a space on the graph between each two possible values. Discrete data often involves counts, ratios, percentage or frequency of characteristics (male/female, pass/fail, small/medium/large, and so on), population data, or data from questionnaires or surveys. Encourage students to think of other situations that must be represented by discrete data. Have students work in groups to count and graph a discrete situation to the class. Emphasize that dots are never connected on discrete data graph.

Discuss the concept of *continuous* data in using examples such as the temperature during the day, miles per hour, and fuel consumption. Display a continuous graph and lead a discussion that emphasizes the importance of the connection of the data points and the information that can be found by inspecting the graph between the data points. Continuous data can be any value(s) on a continuous interval. Continuous data is often associated with physical measurement where values depend on the accuracy of the measuring tool. The scale can be infinitely subdivided into smaller units.

Have students work in groups to create and display different continuous and discrete situations to the class. A good example of the difference would be shoe size versus foot length. Shoe size involves discrete data, but the length of students' feet is continuous data.

Review the water cycle. Place beakers (well-marked with metric measures) of water with thermometers around the room in both sunlit and shaded areas. Assign a group of students to monitor each beaker and to record the level of the water in the beaker and temperature of the water each day. On the fifth day, have students create a chart, graph the data, and explain their findings to the class. Lead a discussion of the differences in the findings, and explore possible hypotheses about the temperatures based on locations of the beakers. (This part of the activity will only work for those classrooms with windows.)

Have students work in groups of 4 to write a word problem that involves continuous or discrete data using a *story chain* ([view literacy strategy descriptions](#)). The first student will initiate the story by writing the first sentence of the problem. The second student adds a second sentence, and the third student provides the last sentence. The fourth student solves the problem. Each *story chain* solution should require the use of a line graph or scatter plot. Students should record their story and solution on construction paper. When complete, groups will discuss their *story chain* with the class. (Example: Student 1 – “Mary left her house and walked to the corner store.” Student 2 – “At the store, Mary met her friend Tabitha, and they bought sodas and a snack.” Student 3- “Mary and Tabitha sat outside the store to enjoy their refreshments.” Student 4 – “The girls decided to race back to Mary’s house to watch tv.”)



Activity 8: Probability Using Spinners (GLEs: 37, 38)

Materials List: Spinner BLM, pencil, paper clips, paper

Make a fair spinner (spinner with all sections exactly the same size) with numbers 1 through 9 and duplicate one Spinner BLM for each pair of students on card stock. Have students cut out the spinner. Show them how to spin a paper clip around a pencil point to make the spinning device. (If available, a graphing calculator can be used to make a fair spinner with the number 1 through 9.)

Ask students to study the numbers on the spinner and have them predict the probability of spinning a multiple of 3. Have the students discuss in their groups how they would collect data to determine the accuracy of their predictions. Have students record the steps in data collection that their group will follow to collect the data.

Ask students to conduct the experiment and record the data. Have each group find the probability for spinning a multiple of 3 from the data they collected.

Discuss *experimental* (data collected and probability figured from collected data) and *theoretical* (the possibilities of each event happening in theory) *probability*. Be sure to include a discussion of why the probability might be different when data is collected through an experiment.

Activity 9: Probability Using Markers (GLEs: 37, 38)

Materials List: brown lunch bags, 10 marbles, markers, or plastic chips for each pair of students (5 blue, 3 red, and 2 green), paper, pencil

Make bags of 10 marbles, markers, or plastic chips (i.e., 5 blue, 3 red, and 2 green). In this activity, students will predict the number of marbles/markers of each color in the bag and compare their prediction with the theoretical probability of drawing each color.

Have students work in pairs. Instruct Student 1 to draw 1 marble/marker from the bag (without looking), record the color, and then replace it in the bag. Repeat this process 10 times. Have Student 2 complete the same process.

Ask, “Using your data, which color marble is most prevalent? Discuss the students’ predictions.” Have the students predict all the colors in the bag based on their data. Discuss their predictions. Ask, “Do you think it is possible to have a color in the bag and never draw that color?” Have students open the bag, then count and record the number of each color. Ask students to compute the experimental probability of drawing each color in the bag based on their data and then to compute the theoretical probability. Ask, “How do these compare?”

Activity 10: Sums Game (GLEs: 37, 38)

Materials List: Sums Game BLM, pencil, brown lunch sacks, 8 same color markers or plastic chips for each pair of students, math learning log

Make sacks containing 8 same color markers or plastic chips for each pair of students. The markers or plastic chips should be marked A-1, B-1, C-2, D-2, E-3, F-3, G-4, H-4. Distribute sacks to each pair of students. *Tell them not to look inside the sack.* Tell them they will play a game involving random draws from the sack, replacing the markers after each draw. Have a whole class discussion about ways to insure that draws are random. Write these ideas on the board or on chart paper.

Go over the directions with the students. Directions: Player 1 randomly draws 2 markers from the sack, computes the sum of the marker numbers, and writes the letters that are on the markers below the sum of the markers on the score card. Replace the markers in the sack, and shake the sack. Player 2 repeats this procedure, recording on a separate score card. Players continue alternating turns. The winner of this game is the *first person* to obtain each different sum at least once *or* to obtain any single sum 6 times.

Example: Player 1 pulls out C-2 and H-4. The sum of 2 and 4 is 6.

Example of score card:

Sums:	2	3	4	5	6	7	8
Combinations:					C, H		

Have students play the game and record their sums. Have the pairs of students post their data on the wall. Give students 5 minutes to walk around the room and make observations from the score cards posted.

Have the students determine a method of determining the *theoretical probabilities*. Theoretically, on any draw from the sacks, what sum is most likely to occur? Least likely? Make an organized chart showing all the possible sums and how each can occur. Then make one or more graphs showing all the possible sums. Given below are two types

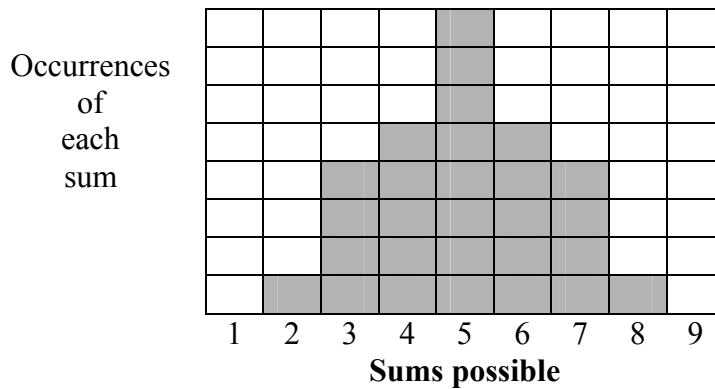
of charts and a bar graph showing the possible sums. Students should come up with something similar to these.

Sums:	2	3	4	5	6	7	8
Combinations:	A,B	A,C A,D B,C B,D	A,E A,F B,E B,F C,D	A,G A,H B,G B,H C,E C,F D,E D,F	C,G C,H D,G D,H E,F	E,G E,H F,G F,H	G,H

Combinations (Sums)						
A,B (2)						
A,C (3)	B,C (3)					
A,D (3)	B,D (3)	C,D (4)				
A,E (4)	B,E (4)	C,E (5)	D,E (5)			
A,F (4)	B,F (4)	C,F (5)	D,F (5)	E,F (6)		
A,G (5)	B,G (5)	C,G (6)	D,G (6)	E,G (7)	F,G (7)	
A,H (5)	B,H (5)	C,H (5)	D,H (6)	E,H (7)	F,H (7)	G,H (8)

The above chart can easily be seen as a bar graph which is shown below.

Theoretical Outcomes



Have students respond to the following prompt in their math *learning logs* ([view literacy strategy descriptions](#)).

Suppose you make one more draw from the sack. Which sum do you think will be result? Justify your thinking with math and examples from your experiment. Allow students to share their entries with a partner as they listen for accuracy and logic.

Activity 11: Fundamental Fun! (GLE: 36)

Materials List: paper, pencil

Have students work in groups of 4. Collect one different object from each member of the group (pencil, eraser, coin, key, ring, and so on). Have each group make all the possible arrangements with the 4 objects contributed by their members. Let students develop their own ways to organize and collect the data (the arrangements they build). Allow them to record the data by listing the objects by name, drawing a sketch, or making a chart. After everyone has finished making all possible arrangements, discuss the methods each group decided to use to record the arrangements as a class. Did you find special ways to make the arrangements? How do you know you have all possible arrangements? Should all the groups have the same number of arrangements? Have a discussion of how arrangements are used in daily life (student ID numbers, telephone numbers, license plate numbers, etc.). Discuss the function and application of the fundamental counting principle.

Have students describe situations where the fundamental counting principle can be used to determine the number of ways an event can occur. Have them use the fundamental counting principle to find the number of outcomes or ways the event can occur.

Activity 12: Relationship of Length of Sides to Area of Polygons (GLE: 41)

Materials List: square color tiles, paper, pencil, additional regular polygons (optional), graph paper

Give groups of students several square color tiles, and ask them to create a rectangle of dimension 1×3 and find the area. (*3 sq units*) Have students double the length of one side and make a rectangle using those dimensions. After students have formed their rectangles, ask students to indicate the lengths of the new rectangle. (2×3 or 1×6) This is a good way to check to see if students understood the instructions. Once this has been clarified, ask students to determine the area of the new rectangle and indicate how it compares to the original (*6 square units, area is twice as much as the original*). Ask students to repeat the process, but this time to multiply one side by 3. (*New rectangles could be 3×3 or 1×9 , and the new area would be 9 square units which is 3 times the original area*). Ask students to predict what would happen if one of the side lengths were multiplied by 4. Students should recognize a pattern that when one side of a rectangle is multiplied by a factor of n , the area of the new figure will be n times as large as the original area.

Ask students to predict how the area would be affected if one side of the original rectangle were multiplied by 2 and the other by 3, and then allow students to test their theories. Provide more examples for students to test until they recognize that the area of the new rectangle is the product of area of the original rectangle and the factors used to generate the new rectangle.

Provide students with problems in which they are given the area of a figure and an indication of how the figure has changed so that they can apply what they learned.

Examples:

A rectangle has an area of 12 square units. The length of one side is multiplied by 4. What is the area of the new rectangle? (*48 square units*)

A rectangle has an area of 10 square units. The length is multiplied by 2 and the width is multiplied by 3. What is the area of the enlarged rectangle? (*$10 \times 2 \times 3 = 60$ square units*)

The area of a square is 9 square units. If the sides are doubled in length, what is the area of the enlarged square (*$9 \times 2 \times 2 = 36$ square units*)

The area of a rectangle is 45 square units. It is enlarged and the new area is 270 square units. Indicate how the sides might have been changed for this to happen. (*1 side was multiplied by 6; one side was multiplied by 2 and the other side multiplied by 3*)

Sample Assessments

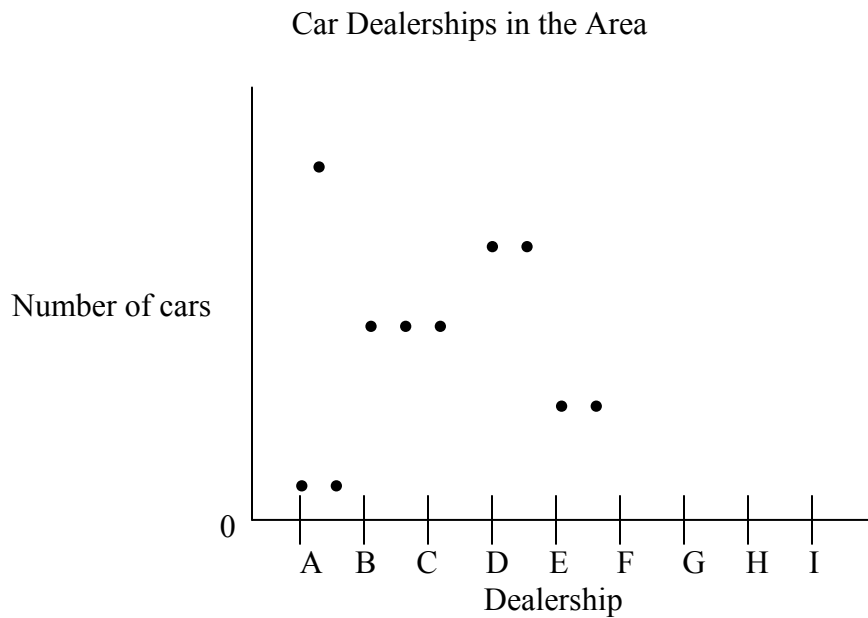
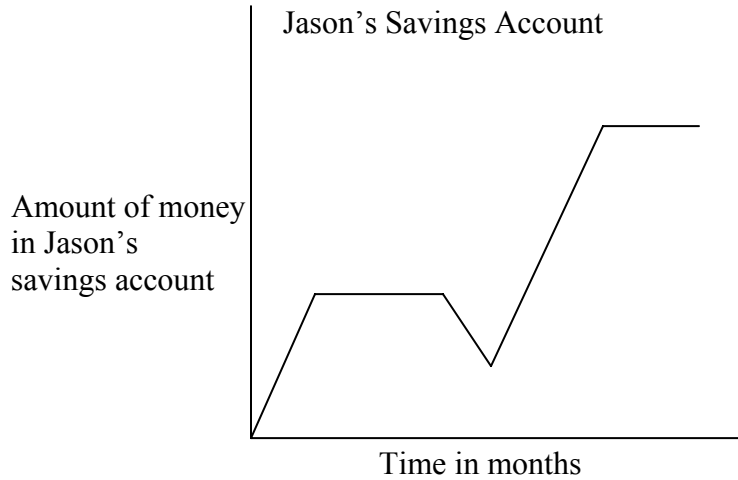
General Assessments

- Determine student understanding as the student engages in the various activities.
- Whenever possible, create extensions to an activity by increasing the difficulty or by asking “what if” questions.
- The student will be encouraged to create his/her own questions.
- The student will use a model to demonstrate an understanding of area.
- The student will create a portfolio containing samples to demonstrate understanding developing a survey on a current issue, collecting data, and creating a graph of their findings.
- The student will complete journal entries by responding to prompts such as these:
 - Explain the purpose and usefulness of circle graphs
 - Explain how an outlier affects the mean in a set of data
 - Explain how doubling or tripling the sides of a square affect the area.

Activity-Specific Assessments

- Activity 1: The student will correctly work the following problem:
The students in Mrs. Chance’s 7th grade math class recorded the following scores on their last math test. Organize the data and describe any patterns, clustered data, gaps, or outliers in the data. Test scores: 77, 76, 97, 90, 95, 89, 76, 100, 95, 77, 90, 77, 96, 77, 88, 97, 88, 97, 33, 96, 100, 97, 100

- Activity 7: The student will tell if the graphs below are continuous or discrete based on the data provided and explain how he/she knows.



Solutions: Graph 1 is continuous data as Jason can save or spend any amount of money over time. Graph 2 is discrete data the number of cars would have to be a whole number.

- Activity 9: The student will write the theoretical probability of each player's winning a game and discuss the fairness of the game given the information below:

Imagine that the following 12 squares are cut apart and placed in a container, and Player 1 and Player 2 play a game by selecting squares. Each player in turn takes a square, records its color, and returns it to the container. Player 1 wins by selecting a red and Player 2 wins by selecting something other than red.

Blue	Red	Blue	Red
Red	Yellow	Red	Yellow
Yellow	Blue	Green	Red

Solutions: 5 out of 12 red squares possible for Player 1 to win (about 42% if they give the percent): 7 out of 12 for Player 2 to win (about 58% if they give percent). It is not a fair game because each player does not have an equal chance of winning.

Grade 7
Mathematics
Unit 5: Angles and Circles

Time Frame: Approximately four weeks



Unit Description

This unit provides practice in graphing and making translations and reflections in the coordinate plane. The relationships among radius, diameter, circumference, and area of a circle are examined. An understanding of angle relationships in triangles is developed. Simple if-then statements are used in problem solving.

Student Understandings

Students can plot points and determine vertices of a figure that has been translated or reflected on a coordinate grid. Students can use the triangle angle sum property and draw and measure angles using a protractor. They apply the formulas for circumference and area of a circle. Students also use logic and symmetry in solving problems. They can explain the effects of a scale factor on the perimeter and area of a figure and apply these in problem solving situations.

Guiding Questions

1. Can students plot points on a coordinate grid?
2. Can students find the coordinates of reflected and translated figures on a coordinate grid?
3. Can students illustrate the relationships between a circle's circumference and area and the measures of its diameter and radius?
4. Can students measure angles to the nearest degree with a protractor?
5. Can students identify and apply the angle-sum relationship for a triangle in problem-solving situations?
6. Can students use their geometric knowledge in generalized problem solving?
7. Can students use elementary logic to solve problems?

Unit 5 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
5.	Multiply and divide positive fractions and decimals (N-5-M)
10.	Determine and apply rates and ratios (N-8-M)
Geometry	
24.	Identify and draw angles (using protractors), circles, diameters, radii, altitudes and 2-dimensional figures with given specifications (G-2-M)
25.	Draw the results of reflections and translations of geometric shapes on a coordinate grid (G-3-M)
26.	Recognize π as the ratio between the circumference and diameter of any circle (i.e., $\pi = C/d$ or $\pi = C/2r$) (G-5-M)
27.	Model and explain the relationship between perimeter and area (how scale change in a linear dimension affects perimeter and area) and between circumference and area of a circle (G-5-M)
28.	Determine the radius, diameter, circumference, and area of a circle and apply these measures in real-life problems (G-5-M) (G-7-M) (M-6-M)
29.	Plot points on a coordinate grid in all 4 quadrants and locate the coordinates of a missing vertex in a parallelogram (G-6-M) (A-5-M)
30.	Apply the knowledge that the measures of the interior angles in a triangle add up to 180 degrees (G-7-M)
Data Analysis, Probability, and Discrete Math	
35.	Use informal thinking procedures of elementary logic involving <i>if/then</i> statements (D-4-M)
Patterns, Relations, and Functions	
40.	Analyze and verbally describe real-life additive and multiplicative patterns involving fractions and integers (P-1-M) (P-4-M)
41.	Illustrate patterns of change in length(s) of sides and corresponding changes in areas of polygons (P-3-M)

Sample Activities**Activity 1: Graphing Ordered Pairs (GLE: 29)**

Materials List: transparencies, overhead, grid paper, pencils

Show students a number line on a transparency that includes positive and negative integers. Ask students to locate/graph several numbers on the number line. On a second transparency, copy the first number line and rotate the transparency to create a four quadrant coordinate grid with x -axis and y -axis. Lead a discussion about the coordinate grid and its parts (x -axis, y -axis, quadrants, origin, and ordered pairs). Graph several examples stressing to find the x -coordinate first, then the y -coordinate.

Plot several points in each quadrant, and have students name the ordered pairs. Next, give the students grid paper; instruct students to create a coordinate grid. Give students several ordered pairs, and have them plot the given points. Allow students to exchange papers and check graphed points.

Activity 2: Battlefield (GLE: 29)

Materials List: Battlefield BLM (2 for each student), pencil

Introduce this game on the overhead. Working in pairs, have students play *Battlefield*, a game in which each player attempts to locate his opponent's hidden targets. Provide each student with two copies of the Battlefield BLM. Instruct each student to use one BLM to keep track of where his/her tanks, trucks, and jeeps are placed, along with the opponent's hits or misses. Students will use the other BLM to track their hits or misses on the opponent's battlefield.

Each player has 5 tanks (each tank occupies 5 consecutive points), 4 trucks (each truck occupies 4 consecutive points), and 3 jeeps (each jeep occupies 3 consecutive points). Instruct each player to locate his/her tanks, trucks, and jeeps on his Battlefield BLM. Have students take turns calling out ordered pairs in an attempt to locate an opponent's targets. Declare a hit or a miss for each point called, recording an O or X on the Battlefield BLM. The game is won by the player with the most hits after a given number of rounds.

Activity 3: Reflections and Translations (GLEs: 25, 29)

Materials List: Transformations-Reflections BLM, pencils, Transformations – Translations BLM, optional materials (plain sheets of paper, scissors, mirrors, graph paper.)

Provide students with three coordinates for the vertices of a parallelogram, B(2,4) C(6,4) D(6,2), and have them plot the points on the Transformations – Reflections BLM. Have the students find and plot the fourth vertex to complete the parallelogram. Discuss the different methods students used to find the missing coordinate. *Answer: A(2,2)*

Next, have students reflect the polygon across the x -axis. If students have trouble, instruct them to trace the parallelogram and cut it out. Now they have a manipulative they can actually “flip” over the axis. Mirrors or folding of graph paper along the axis may also be used to help students understand reflections across the x - and y -axis, if needed. Ask students to determine the coordinates of the vertices of the reflected polygon. *Answer: A'(2,-2) B'(2,-4) C'(6,-4) D'(6,-2).*

Starting from the original position, have students reflect parallelogram ABCD over the y -axis. *Answer: A'(-2,2) B'(-2,4) C'(-6,4) D'(-6,2).*

Repeat this activity with several different polygons. Discuss the different methods students used to find the new coordinates. Instruct the students to write an accurate summary or GISTing ([view literacy strategy descriptions](#)) for reflecting objects over each axis. This is a modified GISTing because there is no text for students to paraphrase; however, students should be able to express in their own words and use math symbols to demonstrate their understanding. For example a student might write, “When a figure is reflected over the x -axis, the x -coordinate stays the same and the y -coordinate becomes the opposite (sign). $(3,2)$ reflected over the x -axis becomes $(3, -2)$.”

Begin the GIST by reminding students of the fundamental characteristics of a summary or gist by placing these on the board or overhead:

- Shorter than the original text
- A paraphrase of the author’s words and descriptions
- Focused on the main points or events

(There is a place on the Transformations-Reflections BLM at the bottom for students to record their *GIST*.)

Provide students with the coordinates of the vertices of a polygon, $[A(2,2) B(2,4) C(6,4) D(6,2)]$, and have them plot the points on the Transformations-Translations BLM. Have students translate the polygon 5 places to the right and up 3 places, and then write the new coordinates of the vertices of the translated polygon. *Answer: $A'(7,5) B'(7,7) C'(11,7) D'(11,5)$.*

Repeat this activity with the original polygon, moving it into each of the four quadrants, and repeat with several different polygons. If students are having a difficult time translating the figures, provide them with polygon shapes to trace on grid paper, translate, and trace in the new position to determine the coordinates.

Discuss the different methods students used to find the new coordinates. Instruct the students to write a *GIST* for translating objects. (There is a place on the Transformations-Translations BLM at the bottom for students to record the rule.)

Activity 4: Area vs Perimeter (GLE: 27)

Materials List: Area vs Perimeter BLM, 1 cm grid paper, scissors, glue or tape, paper, pencil, math learning log

The Area vs Perimeter BLM should be copied and passed out to students for students to complete. The BLM contains a modified *opinionnaire* ([view literacy strategy descriptions](#)) to challenge students’ understanding of area and perimeter. *Opinionnaires* force students to take a position and defend it. Invite students to share their answers with the class.

Pass out 1 cm grid paper and scissors to the students. Have the students cut out rectangular shapes that have an *area* of 24 cm and glue the rectangles on a piece of paper.

Ask them to write the dimensions of each rectangle and to find the perimeter. Have students make a chart to record the dimensions, area and perimeter. Emphasize that perimeter is linear and area is measured in square units. Have students respond to the statement:

All of these rectangular shapes have an area of 24 square cm, but the perimeters are different.

Discuss as a group.

Pass out additional 1 cm grid paper, if needed. Have the students cut out rectangular shapes with a *perimeter* of 24 cm and glue them to a sheet of paper. Ask them to write the dimensions of each rectangle and find the area. Have students make a chart to record the dimensions, area and perimeter. Have students respond to the statement:

All of these rectangular shapes have a perimeter of 24 cm, but the areas are different.

Discuss as a group.

Lead students to recognize that when areas vary---the most elongated rectangle has the smallest area, and the rectangle closest to a square has the largest area. Have a class discussion of the importance of knowing how to compute largest area from smallest perimeter (building a house or garden) or the smallest area with the largest perimeter(dinner party seating guests at tables).

Have students return to their *opinionnaire* to adjust their answers and reasoning as needed. Give students the opportunity to discuss their reasoning with a peer.

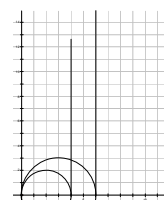
In their math *learning log* ([view literacy strategy descriptions](#)), have students respond to the following prompt: Explain how it is possible for two rectangles to have the same area but different perimeters. Use a model as part of your explanation.

Activity 5: π ! (GLEs: 5, 10, 26, 28, 40)

Materials List: 3-5 different sized lids per group (place a mark on the edge of each lid with a permanent marker to indicate a *starting point*), 2 sheets of cm grid paper per group, tape, rulers, pencils, Around the Lid BLM, calculator, *Circumference and the Dragon of Pi* (optional)

The purpose of this activity is to have students investigate or discover π . Have students work in pairs for this activity. Distribute three to five different-sized lids for each group, two sheets of cm grid paper, and rulers. Have students tape the two sheets of grid paper together. ($8\frac{1}{2}$ in edge to $8\frac{1}{2}$ inch edge to make the paper 22" long). Draw an *x*-axis along the bottom and a *y*-axis along the left edge to represent quadrant I of a coordinate plane.

Instruct students to place each lid on the x -axis so that the diameter of the lid rests along the x -axis, and the outside edge of the lid rests on the *origin*. Ask students to mark the endpoint of the diameter and then draw a line through that point which is perpendicular to the x -axis. (Note: Depending on the length of the diameter, this may or may not be one of the vertical lines on the grid.) Have students trace around the lid showing the semi-circle along the x -axis.



Next, instruct students to place the point marked as the *starting point* on the x -axis at the point where the end of the diameter of the lid is marked. Have the students roll the circle along the perpendicular line that they drew earlier by first placing the *starting point* at the intersection of the x -axis and the perpendicular line. When the *starting point* rotates back to the vertical line, have students mark this point.

Discuss the fact that this one rotation is called the *circumference* of the circle. Have the students find the lengths of diameter (marked on the x -axis) and circumference (marked on the perpendicular line) by reading the units on the grid and record their answers on Around the Lid BLM. Repeat the process with all lids.

Lid Number	Diameter	Circumference	Ratio $\frac{C}{d}$	Decimal Value

Allow students to use a calculator to compute the ratio of the circumference to the diameter as a decimal correct to five places and place their answers in a table. Once students have completed the table on the Around the Lid BLM, ask them to record three observations for the data in the chart.

Lead a discussion concerning the students' findings and the relationship between the circumference and diameter of a circle. Have students see that the pattern is multiplicative and that the decimal ratios are close to the value of π . Make sure that students understand that the relationship between the circumference and diameter indicates that the circumference is a little more than three times the length of the diameter. After the discussion, have the students calculate the average ratio for all five.

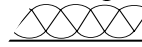
For a closure to this activity, read the book *Circumference and the Dragon of Pi* to the class to reinforce the concepts of pi, circumference, diameter, and radius. An alternative method is to begin the activity, read up through page 13, do the activity, and then finish reading the book.

Activity 6: Understanding Area of a Circle (GLEs: 28, 35)

Materials List: circular objects, ruler or straight edge, scissors, pencil, paper, tape

Using the circular objects from Activity 4, have the students work in groups to approximate (estimate) the area of the circle. Do not let the students look up the formula! Discuss their methods for estimating. (Possible method: Draw a square around the circle. Measure the diameter and use it as the side length of the square.)

Have the students cut a paper circle into eight sectors, and have students position the sectors of the circle as shown to the right.



Lead students to understand that the shape formed is beginning to look like a parallelogram. You may want to have the students cut each of the eight pieces in half and use the resulting sixteen pieces to reform the parallelogram shape. The smaller the pieces, the more the shape will look like a parallelogram. Ask students leading questions so that they understand 1) the base of the rectangular shape is $\frac{1}{2}$ of the circumference and 2) the height of the parallelogram is the radius of the circle. Develop the formula for the area of a circle by starting with the formula for the area of a parallelogram, $A = bh$, and substituting $\frac{1}{2}C$ for b and r for the h . This results in the following:

$$A = bh = \frac{1}{2}C(r) = \frac{1}{2}\pi d(r) = \frac{1}{2}\pi(2r)(r) = \pi r^2, \text{ the formula for finding the area of a circle.}$$

Provide students with practice in using the formula.

Activity 7: Real Life Measures (GLEs: 26, 28)

Materials List: a round/circular object for each pair, measuring tapes, paper, pencil, Circles in Real Life BLM

Have a class discussion of the need to terminology and how to measure circular objects. Ask, “What is someone referring to when he/she speaks of 22 inch rims on a vehicle? A 45-inch round table? Where on each of these objects would you measure to verify the information stated?” (*diameter*)

Ask students to work in groups of four for this activity. Give each pair of students a round or circular object and a tape measure. Have the students measure the diameter, radius, and circumference of their object. Record these measurements. Trade objects with the other pair in the group, measure, and record the measurements. Have the groups study and discuss the relationships between the radius and diameter measurements and between the diameter and the circumference measurements. Ask, “Do your measurements have the same relationships that are found in the circles in the previous activity? Why or Why not?”

Have each pair of students use its measurements to find the area of its object, then trade objects with another pair and find the area of the new object. Groups can compare their answers and check for accuracy and logic.

Provide real-life problems in which students find circumference, area, diameter or radius of given objects (e.g., car or truck tires, bicycle tires, gasoline drums, circular pools, silos, tree trunks, water tanks). Additional practice can be found on Circles in Real Life BLM.

Activity 8: Circumference and Area (GLEs: 24, 27, 28)

Materials List: rulers, pencils, compasses, Circumference and Area BLM, computer with internet access (optional)

Have the students recall the activity in the last unit where they doubled the length of the sides of a polygon and then compared the areas and perimeter (Unit 4, Activity 11). Break the students into groups of 4. Give each group rulers and three radii measurements.

Have the students draw three different circles with the given radii. Give $\frac{1}{3}$ of the groups the measurements 4 cm, 8 cm, 16 cm, another third the measurements 2cm, 4cm, 8cm, and the last third of the groups the measurements 3cm, 6cm, 12cm.

Have students complete the Circumference and Area BLM chart with given radii, diameter, circumference, and area and answer the questions that follow.

Discuss as a class the three different sets of numbers used and each group's observations and reasoning.

An example of the chart with some answers is given.

Radius	Area	Diameter	Circumference
2 cm	12.56cm ²	4cm	12.56cm
4 cm	50.24cm ²	8 cm	25.12cm
8 cm	200.96cm ²	16 cm	50.24cm

After discussing each group's observations and reasoning, have students complete this statement: "If the radius of a circle tripled, then ..." Be sure to have students justify their reasoning.

An interactive math lesson for calculating the circumference of a circle can be found at <http://www.aaamath.com/geo612-circumference-circle.html>.

An interactive math lesson to teach the finding the area of a circle can be found at <http://www.aaamath.com/geo612-area-circle.html>.

Activity 9: What's Your Angle? (GLEs: 24, 30)

Materials List: What's Your Angle BLM, protractor, pencil

Give each student a copy of the What's Your Angle BLM. Individually, have students place three points on each circle to act as the vertices of a triangle, and then use the straight edge of a protractor to construct a triangle. Ask students to measure each angle of the triangle using the protractor, and record the measurements.

Have students share by comparing and discussing within their group the types of triangles drawn and the sum of angles of each of their triangles. Have students recognize that the sum of the angles in a triangle appears to be 180 degrees. Because of variances in measurements, this may not be obvious. This concept is revisited in the next activity.

Discuss the term altitude. Have the students draw an altitude in each of the triangles they drew inside the circles. Be sure to emphasize that the two perpendicular sides in right triangles are also altitudes.

For practice, have students draw triangles with specific side lengths and/or specific size angles. Give students the opportunity to draw 2-dimensional figures with specified angular measures.

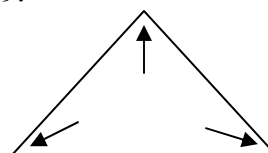
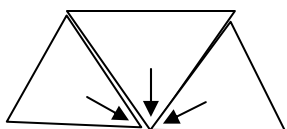
Activity 10: Interior Angles of a Triangle (GLEs: 24, 30)

Materials List: construction paper, protractors, scissors, pencils, tape, math learning log

In this activity the understanding that the sum of the interior angles of a triangle is 180 degrees is reinforced by using a method in which none of the individual angle measures is known. Since this process works for any triangle, it is considered a proof of the triangle sum theorem.

Divide students into teams of three students each. Give each student a sheet of construction paper and a protractor. Each group should also have at least one pair of scissors. Ask each student in the group to draw a triangle with each person drawing a different type triangle (acute, obtuse, right).

Have students cut out their triangles, label each angle with a letter (put the letter in the interior of the triangle near the vertex), and then tear off the angles of the triangle. Instruct students to place the angles adjacent to one another with vertices touching. Ask students how this proves that the sum of the three angles is 180 degrees and how this process differs from what was done in Activity 9.



Next, have each student draw a triangle on construction paper and label two of the angles with measures they find by using the protractor. The third angle should be marked with a question mark. On the back of the triangle, place the numerical value of the missing angle. Have teams exchange their triangles with another team, and then find the missing angle using the triangle sum theorem (rather than measuring the angle). They can quickly check each other's work by comparing their answers to the one written on the back of the triangle. Students should discuss if there are differences of opinion regarding the answers.

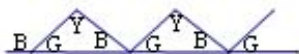
In their math *learning log* ([view literacy strategy descriptions](#)), have students describe how to find the sum of the interior angles of a triangle as well as any new understandings they now have about the angles in triangles.

Activity 11: Triangle Fun! (GLEs: 27, 30, 41)

Materials List: triangles cut from stiff paper, yellow, blue, and green colors, colored pencils or markers, unlined paper, pencils, straight edge.

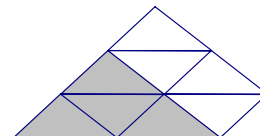
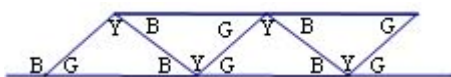
To prepare for this activity, cut triangles from stiff paper. Make sure that the longest side of the triangle used is no longer than 3 inches for this activity. If each student has a different triangle, the activity becomes more powerful.

Give students a triangle cut from stiff paper and have them color each angle of the triangle a different color. In the diagram below, Y is for the color yellow, B is for the color blue and G is for the color green. Instruct students to not label the angles of their triangles.



On a sheet of unlined paper, have the students draw a line with a straight edge near the bottom third of the paper. Take the triangle and place a base of the triangle on the line with the altitude pointing to the top of the paper. Trace it. Color each angle of the triangle drawn on the unlined paper the same as it is on the cut out triangle.

Slide the cut out triangle to the right on the line so that the B vertex and the G vertex just touch, and trace it. Color each angle of the triangle. Slide the triangle over to the left and/or right until the page runs out, trace it, and color in the angles of the triangle that show. Rotate the triangle so that the Yellow vertex will fit snugly into the vertex formed by the Blue and Green angles. Trace this triangle. Color in the angles of the drawn triangle. Slide the triangle over until Yellow fits snugly into the vertex formed by the Blue and Green angles, trace and color in the angles.



Rotate the triangle for the next level, and trace the triangle, color the angles, slide the triangle, trace the triangle, then color the angles. Continue this until you run out of room at the top of the page.

In groups, have the students study the figure and list as many conclusions as they can. Have students outline or highlight certain parts of the figure or cover up certain parts to make their conclusions. Some possible conclusions are as follows: the total degrees of a triangle is 180° (all three different colors along the bottom and the sides come together to form a straight angle or 180°), the sum of the angles in 4-sided figures (squares, parallelograms, rectangles, trapezoids, and rhombii) is 360° , the sum of the angles around a point (a circle) is 360° , doubling the sides of the triangle quadruples the area, doubling the sides of the triangle doubles the perimeter and doubles the original altitude, and a doubled triangle is similar to the original triangle. If students were able to make enough copies of the triangle to create a tripled triangle, discuss the effect of tripling on area, perimeter, and altitude with the class. Make sure that students show proof of their findings by showing where the property is illustrated in the drawing. Use probing questions to help students draw conclusions that were not addressed.

Activity 12: If, Then! (GLE: 35)

Materials List: paper, pencils, large paper or newsprint, markers

Get students ready to create mathematical if-then statements by having them change some everyday statements into if-then statements. For example, "I am going shopping if I get paid today" can be rewritten as "If I get paid today, then I am going shopping." Use several other common situations before proceeding to the mathematical if/then statements. Next, provide students with several geometry facts, such as the sum of the angles of a quadrilateral is 360 degrees. Now, challenge students to create if/then statements that are equivalent to these facts. For example, if a polygon is a quadrilateral, then the sum of the measures of its interior angles is 360 degrees. If a quadrilateral is regular, then each angle measures 90 degrees. Also, have students create if/then statements to illustrate the relationships that exist among the circumference, diameter, radius, and area of a circle. For example, if C is the circumference of a circle, then the diameter of the circle equals C divided by π .

Have students work in pairs to create 5 if/then statements that relate to this unit. After 10 minutes, have each pair exchange with another pair. Students should check each other's work for understanding and misconceptions. After 5 minutes the two pairs who exchanged should form a group to discuss the 10 statements and choose 2 or 3 to share with the class. These may be written on large paper or newsprint. Have a class discussion about the statements.

Activity 13: More If, Then Statements (GLE 35)

Materials List: More If, Then Statements BLM, pencils

Give each student a copy of More If, Then Statements BLM, and have them work to complete each if, then statement. Remind students the information they provide must be mathematically based. After students have been given time to complete the statements, have them pair with another student to compare and discuss the statements. Allow groups time to share their statements with the class. Use the students' shared statements to clear up any misconception for this unit. Have the students complete additional if, then statements as needed.

Sample Assessments

General Assessments

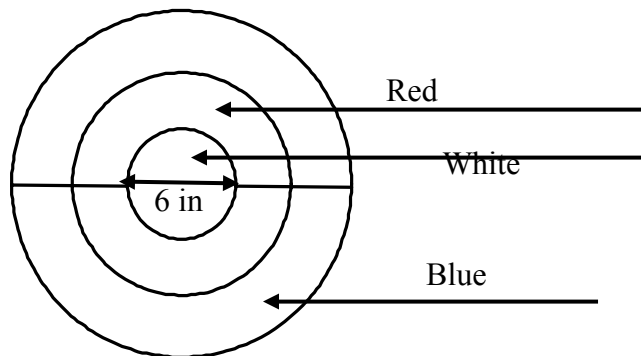
- Determine student understanding as the student engages in the various activities.
- Create extensions to an activity by increasing the difficulty or by asking “what if” questions.
- The student will be encouraged to create his/her own questions to evaluate his/her understanding of circle concepts.
- The student will create a portfolio containing samples to demonstrate understanding of one of the concepts in the unit. The following is an example:
Anne called The Pizza Man to order a 12-inch pizza for her and her best friend. When the pizza delivery boy came to the door with her pizza, he had two boxes. He told Anne that they had run out of twelve-inch pizza boxes and the owner had told him to deliver two, six-inch pizzas instead. He told Anne that it was the same amount of pizza and each of them would have her own pizza. Anne disagreed with the pizza boy and convinced him that she was correct. Explain how you think Anne might have proven this to the pizza boy. Use diagrams and /or mathematics in your proof.
- The student will trace, measure and record the diameter of three circular objects and then find the circumference and area of each.
- The student will complete journal writings using such topics as:
 - Explain the difference between circumference and perimeter.
 - Form a conjecture about the relationship between the shape of rectangles and their areas and perimeters.

Activity-Specific Assessments

- Activities 1 and 2: The student will work the following problem correctly:
 Kayla plotted the following points or triangle ABC on a coordinate grid:
 A (2,5), B (6,3), C (4,8) Stacey reflected the triangle across the x-axis.
 Find the coordinates of the corresponding points on the reflected triangle.
Solution: The new points would be A' (2 , -5), B' (6 , -3) , and C'(4, -8).
- Activity 3: The student will work the following problem correctly:
 If you are going to fence in a dog run with an area of 180 square feet, what shape and dimensions would you choose? Why? Fencing costs \$5.95 per foot. How much would the fencing cost? *Solution: A dog run should be long and narrow. The dimensions could be 5 feet by 36 feet for the dog run. The perimeter would be 82 feet. The cost would be $82 \times \$5.95 = \487.90 .*

If you wanted to build the same area with the smallest perimeter for your dog pen, what dimensions would you choose? Why? How much would the fencing cost here? *Solution: The more square you get for the figure, the less perimeter you have. The dimensions should be 15 ft by 12 ft. for the smallest perimeter with an area of 180 square feet. Perimeter is 54 feet. $54 \times \$5.95 = \321.30*

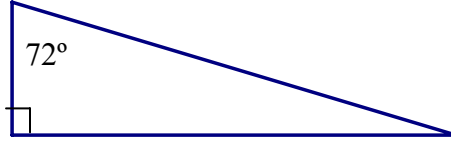
- Activity 7: The student will work the following problem correctly:
 Randy’s dad cuts circular targets for archery practice. He asked Randy to help him by cutting one circle with a diameter of 12 inches out of red plastic and a second circle with a diameter of 18 inches out of blue plastic. He wanted a white center bull’s eye circle cut with a diameter of 6 inches. Randy was to glue these circles together into a target that looked like the picture. Determine the area of each color on the target, and explain your method of solving the problem.



*Solution: The white bull’s eye has an area of 28.26 sq in
 The red area that is showing has an area of 84.78sq in
 (Whole red circle area of 113.04 – white circle area of 28.26)*

The blue area that is showing has an area of 141.3 sq in
(Whole blue area circle of 254.34 – whole red circle of 113.04)

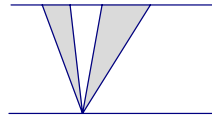
- Activity 9: Jordan rides his skateboard over a ramp shown below. What is the measure of the angle that the ramp makes with the ground?



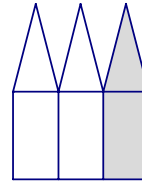
Solution: The angle measures 18°

- Activity 10: The student will find the sum of the measures of the angles of each shaded region and explain his/her reasoning.

a)



b)



Solutions: a) 360° b) 540°

**Grade 7
Mathematics
Unit 6: Measurement**

Time Frame: Approximately four weeks



Unit Description

This unit extends the work with measurement and the application of perimeter and area concepts to irregular and regular polygons. Comparisons between metric and U.S. measures for area, weight/mass, and temperature are made.

Student Understandings

Students work with applying the relationship between length of sides and perimeters and areas for similar polygons, extending the work with patterns noted in previous units. Finally, students compare metric and U.S. measures for area, weight/mass, and temperature. They will convert, within the *same system*, between units and compare measurements between the systems using common reference points.

Guiding Questions

1. Can students convert between measures of area within the same system of measurement?
2. Can students work with the changes in scale and the changes in perimeter and area?
3. Can students compare the relative measures of area and weight/mass across different measurement systems?
4. Can students compare the relative measures of temperature across different measurement systems?
5. Can students apply these understandings in problem-solving situations?

Unit 6 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
5.	Multiply and divide positive fractions and decimals (N-5-M)
7.	Select and discuss appropriate operations and solve single- and multi-step, real-life problems involving positive fractions, percents, mixed numbers, decimals, and positive and negative integers (N-5-M) (N-3-M) (N-4-M)

Measurement	
20.	Determine the perimeter and area of composite plane figures by subdivision and area addition (M-1-M) (G-7-M)
21.	Compare and order measurements within and between the U.S. and metric systems in terms of common reference points (e.g., weight/mass and area) (M-4-M) (G-1-M)
22.	Convert between units of area in U.S. and metric units within the same system (M-5-M)
23.	Demonstrate an intuitive sense of comparisons between degrees Fahrenheit and Celsius in real-life situations using common reference points (M-5-M)
Patterns, Relations, and Functions	
40.	Analyze and verbally describe real-life additive and multiplicative patterns involving fractions and integers (P-1-M) (P-4-M)

Sample Activities

Activity 1: Calculate Perimeter and Area of a Plane Figure (GLEs: 5, 7, 20, 22)

Materials List: video of home improvement show and TV or projector (optional), House Plan BLM, newsprint, color markers, rulers, catalogs or sale papers from home improvement stores, scissors, glue and/or tape, paper

Allow students to view a clip from a home improvement show for motivation. Tell students that when beginning various home improvement projects, homeowners often must know the dimensions of each room. Using the House Plan BLM or another simple house plan that includes several rooms composed of composite plane figures with a scale of $\frac{1}{2}$ -inch equals 5 feet (this can be changed to suit the project), have a class discussion on how to measure and find the dimensions of the floors in a room on the house plan. Review how to find the area of the walls of the rooms with the assumption that the walls are 8 feet tall. Have a discussion about how to convert between units of area. Tell students that the area of a kitchen is 75 square yards, and is being covered by tiles that measure 1 square foot. How many square foot tiles are needed to cover the floor? *225 tiles or 225 square feet*

Working in groups of four, have each group pick a room that they want to redo. Have them draw a scale model of the room on newsprint and provide a key for the scale used. Then have students find the perimeter and area of the walls, floor and ceiling of the room. Instruct students to make clear explanations of the processes used at each phase of the problem. Using catalogs from home improvement stores, have students select molding, flooring, paint, wallpaper, and other improvement items. Have students cut out pictures and prices of materials, glue them to a sheet of paper, and then calculate the cost for each. To facilitate evaluation, have students show their work on the paper next to the pictures.

Allow students to decorate by cutting out pictures to show how they want each room to look.

This project will take several days depending on the number and type of improvements incorporated. Include area unit conversions within the same system, U.S. or metric, in the activity having students show any conversions on their papers. For example, if carpeting is sold by the square yard and the living room has an area of 200 square feet, how many square yards of carpet are needed? Students should include the cost per improvement and a total cost for the room.

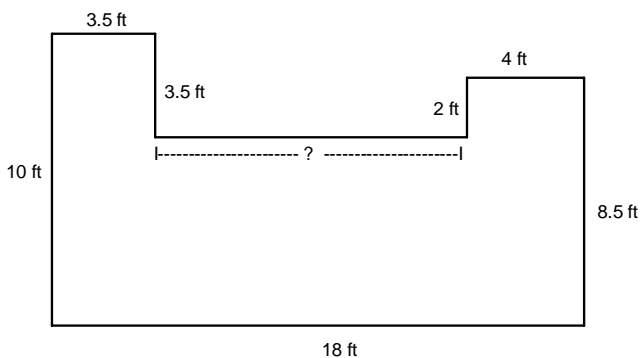
After each group has finished, have them assume the role of *professor know-it-all* ([view literacy strategy descriptions](#)) and present their room remodeling to the class, explaining how they figured the cost of each improvement. Remind other students to formulate questions to ask of the know-it-alls and to hold them accountable for the accuracy of their answers. After all presentations have been made, place all the rooms together to build the house. Display on a classroom wall.

Activity 2: Pool and Hot Tub Addition (GLEs: 20, 22)

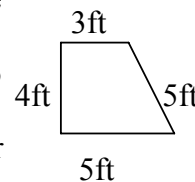
Materials List: Pool & Hot Tub BLM, pencil, math learning log

Pose this situation as an addition to Activity 1, and have students work in groups. The Pool and Hot Tub BLM may be used.

The swimming pool that is to be put in the back yard has an irregular shape as shown below. A pool cover is needed to keep the leaves out this winter. Find the area of the pool. All corners are 90° . Pool covering material costs \$4.95 per square yard; how many square yards will we need, and how much will the pool cover cost? Explain how you arrived at finding the area of the pool and the cost of the pool cover. We also need to know the perimeter of the pool, so that we can buy bricks to go around the edge of the pool. Find the perimeter. Bricks are 6 inches long. How many bricks will we need to buy to put one row of bricks end to end around the pool? Bricks cost 60¢ each. How much will we spend on bricks? Explain and show how you determined the perimeter of the pool, and the cost of the bricks.



A hot tub in the shape of a trapezoid with the dimensions shown will be built along the right side of the pool and adjacent to the bricks. A top view of the hot tub is shown. Find the cost of making a cover for the hot tub. Since the hot tub will be placed next to the swimming pool, the side with length 4 ft. will not be bricked. Find the cost of bricking the remaining three sides. Show all work for determining the cost of the cover and the bricks.



Ask students to respond to the following prompt in their math *learning logs* ([view literacy strategy descriptions](#)). Depending on the students, there may need to be a similar discussion prior to the students completing the math *learning log*.

Are the answers you found during the Pool and Hot Tub activity realistic? Give at least three reasons to support your answer or describe how the answers should be adjusted to fit the real-life situation.

Example answers: If the pool covering material is sold by the yard, it will have to be sewn together and extra material should be bought. For the bricks around the pool, the end bricks will need to extend the width of the brick to make an even corner. Also, the lengths are not all multiples of 6, and the bricks will need to be broken. Will there be mortar between the bricks? The cover of the hot tub is a trapezoid, so extra material will be needed for the slanted side. These things encourage higher order thinking.

Activity 3: Measuring Scavenger Hunt (GLE: 22)

Materials List: a list of measurements describing various objects (several for each pair of students), yard sticks and/or measuring tapes, meter sticks, protractors, pencil, paper, Measuring Scavenger Hunt BLM

Make a list of measurements describing various objects found in the classroom or in a specified area outside on the school grounds. Provide measures in the U.S. and metric systems with angle measurements included. Be sure to include descriptions that are given in units of area. Measurements may be added to the Measuring Scavenger Hunt BLM and given to students to find the objects. Give each pair of students a list of objects, a yard stick (or tape measure), a meter stick, and a protractor. Have students hunt for each object described, measuring objects to find the ones on the list and writing the name of the object found on paper. Specify a time limit for completion of the hunt.

(Example of description of objects: This object is 6 inches off the ground, and its dimensions are 12 inches by 4 inches, or this object has an angle that measures 60° and the sides that form the angle each have a length of 15cm, or the top of this object has an area of 10 square inches and sits on the ground.)

When the students return to the classroom, have them convert specified measurements within the same system (e.g., 6 inches = _____ foot, 12 in = _____ ft, 2 square feet = _____ square inches, 36 square feet = _____ square yards).

As an extension, allow each pair of students to find and write a measurement description for an object, then swap descriptions with another pair to find the object.

Activity 4: Metric Madness (GLEs: 22, 40)

Materials List: metric rulers, meter sticks, paper, pencil, place value chart (can write on board)

Pass out metric rulers and meter sticks to groups of students. Have the students study the rulers and meter sticks, and write down observations about the relationships between units. Lead a class discussion to help students develop an understanding of equivalencies. Discuss how the metric system is based on 10, the same as the place-value system. Help the students connect the metric prefixes with the decimal place value system. Place metric prefixes with decimal names on a place value chart to help students remember their values (i.e., thousands–kilo, hundreds–hecto, tens–deca, ones–meter, tenths–deci, hundredths–centi, thousandths–milli). Discuss this chart emphasizing the most needed units–kilo, centi and milli. Using the meter stick, have the students convert millimeters to centimeters and kilometers and vice versa. Record each conversion on the board, study the conversions, and discuss how converting from a smaller unit to a larger unit requires division and converting from a larger unit to a smaller unit requires multiplication. Make sure that students are aware that the prefixes work the same with grams and liters.

Extend the activity with explanation, examples, and problems involving converting between measures of area in the metric system.

Example: Joshua is planning to cover a coffee table with material that is sold in square meters. His table measures 60 cm by 32 cm. How many square meters of material should he purchase?

Present additional real-life problems in which students convert between units of area.

Activity 5: Compare U.S. and Metric Systems of Length (GLE: 21)

Materials List: yard sticks, meter sticks, large paper clips, other objects for groups of students to measure (e.g., straw, large index card, textbook, floppy disk, pencil)

Pass out yard sticks and meter sticks to groups of students. Have students compare the lengths of a yardstick and a meter stick, and write down observations about their relationships. Help students identify benchmarks to use to approximate equivalent measurements between the U.S. and metric systems - a yard is a little less than a meter, $2\frac{1}{2}$ cm is about 1 inch, and 5 miles is about 8 kilometers.

Have students measure the width and thickness of a large paper clip to learn reference points for a cm and mm, respectively. Provide several items for students to measure (e.g., straw, large index card, textbook, floppy disk, pencil, height of door, width of door, window). Have each group measure four times and record the measurements on a chart.

Require that students decide the appropriate U.S. measurements—yards, feet or inches—to use when measuring.

Have students measure the same four items using the appropriate metric measurements—centimeter, meter or kilometer.

As a class, discuss the methods they used for measuring the items in the U.S. and metric systems. Compare the measurements from group to group to determine accuracy.

Have students use their sense of the size of common units to estimate measurements (e.g., the height of the classroom door is about two meters) and use commonly understood benchmarks to estimate large measurements (e.g., the distance between the middle school and the high school is about the length of ten football fields).

Activity 6: Compare U.S. and Metric Systems of Capacity (GLE: 21)

Materials List: 6-10 empty containers demonstrating metric and U.S. measures without labels or markings, pencil, paper, sand or water

Provide students with the opportunity to compare measurements between the two systems. Place 6 -10 empty containers labeled as A through J in random size order on a table where students can see them easily. Make sure that all packaging labels have been removed and that some containers demonstrate metric measures and others U.S. measures (e.g. pint milk carton, soda can, $\frac{1}{2}$ gallon juice carton, liter of soda, gallon jug, 2-liter bottle, 5-gallon water jug). Have each pair of students study the containers, and order them from largest to smallest. Instruct students to compare the objects visually and not to touch the containers. Ask students to return to their desks and record their reasons for the order they chose.

After all groups have completed their ordering, have each group present and explain to the class its ordering of the items. Make a list of all the different ordering on the board. Pour sand or water into the containers to show what the actual ordering should be. When moving between the metric and U.S. systems, have students find approximate equivalents to become familiar with the benchmarks between systems—a quart is a little less than a liter.

Activity 7: Compare U.S. and Metric Systems of Weight (GLE: 21)

Materials List: How Do I Compare? BLM, sugar packets, rice, Ziploc[®] bags, masking or electrical tape, items for students to weigh (e.g., textbook, bag of flour, can of soup, bag of chips), digital scale

Instruct students to complete the How Do I Compare? BLM an *opinionnaire* ([view literacy strategy descriptions](#)) about U.S. and Metric systems of weight. It will force the

students to take a position and defend it with reason. Place students in groups of 3 or 4, and have them share their opinions and reasoning with the group. Instruct students to compare their reasoning with the others and debate each other's reasoning.

Put the *opinionnaire* to the side for now. Have students use digital scales to weigh a small restaurant-sized packet of sugar. It should weigh about a gram. Have a Ziploc® bag with about $2\frac{1}{4}$ pounds of rice so that students will have a reference point for a kilogram. Provide several different packages with the weights covered with masking or electrical tape so that they cannot be read (e.g., textbook, bag of flour, can of soup, bag of chips). After looking at the packages, have the students order the items by weight from least to greatest. Allow students to pick up and hold each item to guess the weight in reference to the benchmarks — the weight of the bag of rice and the small package of sugar. Have the students study the order of the items and allow them to reorder any item after reviewing the benchmarks. Because most students have a very poor sense of weight, make sure they always have approximate equivalents to use or become familiar with the benchmarks between systems.

After all groups have completed their ordering, have each group present and explain to the class their ordering of the items. Make a list of all the different ordering on the board. Allow students to use digital scales to weigh the items, and adjust their lists accordingly.

Have students revisit the each statement on their *opinionnaires*. Allow them to adjust their answers and reasoning, and discuss why changes were needed.

Activity 8: Compare U.S. and Metric Systems of Measurements (GLE: 21)

Materials List: pencil, paper

Provide students with a list of situations in which they select the appropriate size and type of unit for a given measurement situation. For each situation, have the students explain their reason for choosing a given unit. For example, they should write that it makes sense to use liters rather than milliliters when determining the amount of soda for the school dance, or that milliliters may be quite appropriate when measuring a small amount of a liquid for a science experiment. Be sure to include situations that allow students to choose between two U.S. measurements.

Have students work in pairs to discuss and defend their choices.

Activity 9: What's the Temp? (GLEs: 21, 23)

Materials List: 3" x $8\frac{1}{2}$ " strips of paper, dual thermometers, cups, water, ice, paper, color markers, large number line

Give each student a strip of paper measuring 3 inches by $8\frac{1}{2}$ inches. Have students write 100°C in the top left hand corner and 0°C in the bottom left hand corner on the $8\frac{1}{2}$ side of the strip. Have the students fold the paper in half, then in half again, and write 50°C on

the left hand side at the half mark fold. Have students fill in the missing degree measures along the other two folds on the left hand side of the strip of paper (i.e., 25°C and 75° C).

On the top right hand corner, have students write 212° F, and on the bottom right hand corner, have them write 32° F. These are two benchmark temperatures students should know, the boiling point and the freezing point of water, respectively. From here, have students fill correct temperatures on the other fold marks. When finished, the labels from top to bottom should be 212° F, 167° F, 122° F, 77° F, 32° F.

Let students use this strip as a reference when comparing Celsius and Fahrenheit temperatures. Have students use a dual thermometer (showing both Fahrenheit and Celsius temperatures) to measure the temperature of various items—a cup of ice water, a cup of room temperature water, and a cup of hot water. Create large temperature “number” lines with labels for both Fahrenheit and Celsius degrees, and have students place drawings or pictures of various items—a hot cup of cocoa, a glass of ice water, a nice spring day, a winter day—on the line to indicate a reasonable temperature for each item.

Activity 10: Temperature in the Newspaper (GLE: 23)

Materials List: newspapers, paper, pencils, math learning log

Bring several weather pages from the newspaper to class (or have the students bring them). Give each group of four students a weather page. Have the students record as many observations as they can in 8 minutes. Discuss the students’ observations--the temperatures and all the other information found on the weather page. Ask, “Are these temperatures in Celsius or Fahrenheit? How do you know?” Have students place and order the Fahrenheit temperatures listed on the weather page on the temperature strip created in Activity 9. Have the students determine the approximate Celsius temperature for each Fahrenheit temperature using the temperature strip. Have them use one of the formulas, $C = \frac{5}{9}(F-32)$ or $F = \frac{9}{5}(C+32)$, to determine how close they were in estimating the correct temperature using the temperature strip.

Students should respond to the following prompt in their math *learning logs* ([view literacy strategy descriptions](#)).

Matthew turned on the radio to hear the weather forecast for today. He heard the weatherman say the high temperature would be 12° Celsius. Will Matthew need his jacket today? Justify your answer using math symbols and diagrams to explain how Matthew can change Celsius temperature into Fahrenheit temperature and vice versa.

Solution: Yes, the high temperature will be 53.6°F. Students should justify their answer by showing how to use the formula $F = \frac{9}{5}(C + 32)$. If solving for degrees Celsius, the formula is $C = \frac{5}{9}(F - 32)$.

Allow students to exchange their *learning log* responses with a partner to check for accuracy and logic.

Sample Assessments

General Assessments

- Create and use checklists to determine the students' understanding of measurement concepts.
- Whenever possible, create extensions to an activity by increasing the difficulty or by asking "what if" questions.
- The student will be encouraged to create his/her own questions to evaluate his/her understanding of measurement concepts.
- The student will accurately measure different objects using a variety of measurement tools.
- The student will draw a scale model of his/her bedroom and create a makeover of the room. The makeover will include the purchase of flooring (carpet, tiles, and so on), paint for the walls and relocation of furniture based on scaled drawings of the pieces of furniture. The student will show all mathematical steps for the work. Work will be placed in the student's portfolio.
- The student will complete journal entries using such topics as:
 1. Explain the difference between measurements reported in ft and ft²
 2. Explain what may have happened if two people had different results when measuring the same item. For example, one person measures a board as 18 feet, and another measures it to be 6 feet.

Activity-Specific Assessments

- Activity 1: Use the example rubric provided in the blackline masters to evaluate the student's understanding of the remodeling activity.
- Activity 2: The student will solve the following problem correctly:

Your backyard is a rectangular shape that is 100 feet by 40 feet. The patio in the backyard is 18 feet by 20 feet. How much of the backyard is not covered by the patio?

- Activity 8: The student will determine an appropriate unit for measuring each of the following and explain his/her choice:
 - a) The capacity of a soda can
 - b) The length of a person's foot
 - c) The parking lot of a school
 - d) The weight of a small box of raisins
 - e) The thickness of a fingernail
 - f) The heat from the stove
 - g) A bag of potato chips
 - h) The area of a classroom
 - i) The area of a leaf
 - j) A single dose of liquid medicine

**Grade 7
Mathematics
Unit 7: Probability**

Time Frame: Approximately four weeks



Unit Description

This unit solidifies basic counting as students compute probabilities from collected data and record the data in tables and charts to help analyze the outcomes of experiments. These experiments are both theoretical and experimental in nature.

Student Understandings

Students should be able to count and determine probabilities from collected data in tables, charts, and displays. Students will make comparisons with theoretical and experimental probabilities.

Guiding Questions

1. Can students use charts and tables to find all possible outcomes which are based on the fundamental counting principle?
2. Can students determine probabilities of the occurrence of an event after listing all possible outcomes for the event?
3. Can students compare and contrast the outcomes associated with theoretical and experimental analyses of the same situation?

Unit 7 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Data Analysis, Probability, and Discrete Math	
32.	Describe data in terms of patterns, clustered data, gaps, and outliers (D-2-M)
36.	Apply the fundamental counting principle in real-life situations (D-4-M)
37.	Determine probability from experiments and from data displayed in tables and graphs (D-5-M)
38.	Compare theoretical and experimental probability in real-life situations (D-5-M)

Sample Activities

Activity 1: It's Fundamental! (GLE: 36)

Materials List: 3 different color books, classroom objects to be arranged, calculators, paper, pencil

Introduce the fundamental counting principle by using three different color books to demonstrate the number of ways the books can be arranged or ordered when placed on a bookshelf. Indicate to students that these arrangements are called permutations. Permutations occur when the order of the individual items is important in determining how many arrangements can be made.

Label each book with a number or letter (A, B, C) to distinguish it from the others. Have students record the order of the books every time one book is moved. Invite students to participate by suggesting how many places are left in the rack as each book is positioned. Lead a discussion of ways in which this concept is used in daily life (e.g., license plates, social security numbers, telephone numbers, student numbers).

Divide students into groups of four and give them different objects to arrange. (These objects can be anything available and calculators may be used.) Tell students, as *professor know-it-all* ([view literacy strategy descriptions](#)), they are to set up a display, label the number of possible placements, and calculate the number of possible arrangements. Each group should write 2 to 3 questions it feels its peers will ask and provide answers to them. When it has finished, each group will share its display and explain the solution to its problem to the class. It will have to defend its reasoning to the class as well as answer any additional questions they may have.

Extend discussion to other (larger) situations, such as counting the number of ways thirty students could line up in a single file.

Activity 2: How Many Choices? (GLE: 36)

Materials List: pencil, strips of paper, box, posters, color markers

Have students *brainstorm* ([view literacy strategy descriptions](#)) about situations where the fundamental counting principle could be used. Instruct them to write each on a strip of paper and place in a box. Break students into groups of 3 or 4, and have each group choose 2 strips, and collaborate on making posters that illustrate using the counting principle with their situations. Instruct students to be ready to share with the class in 20 minutes. Examples of situations:

- There are 4 flavors of yogurt and 10 toppings. How many choices are available if you can have only one topping?

- You can order from 3 sizes of pizza, 2 types of sauces, and 3 types of cheese. How many choices of pizza are there?
- You are going to make a peanut butter and jelly sandwich from 5 types of bread, one kind of peanut butter, and 4 flavors of jellies. How many different kinds of sandwiches can be made if only one flavor of jelly can be used?

Activity 3: Determine Probability from Data (GLE: 37)

Materials List: computer with Internet access, basketballs and basketball net or wastepaper baskets and foam balls, paper, pencil

Introduce the lesson with basketball statistics about national players (<http://www.basketball.com/> or <http://www.nba.com/statistics/index.html>) or players from the local high school. Guide the discussion of the school's team using local newspaper reports or other sources to point out vital statistics (e.g., player with the most 3-point shots completed or most rebounds). Ask students what it means when someone says that a certain basketball player is a 75% free throw shooter. Pick two or three players from the list of statistics, and ask students to determine the probability that the player will make his next free throw. Ask students to explain how to calculate a player's free throw percentage (or his/her probability of making the next free throw). Make sure that students understand that these values are calculated by dividing the number of free throws made by the number of free throws attempted. Ask students to give fractional equivalents of the percentages (i.e., a free throw percentage of 65% means the player makes 13 out of 20 free throws or about 2 out of every 3 free throws).

Have students work in groups of 4. Let students predict the number of free throws they think they can make out of 20 tries. Have students go to the basketball court and let each student shoot 20 times from the free throw line. (Substitute a wastepaper basket or milk crate and a foam ball to create an indoor version of the activity.) Have students take turns recording the data as team members shoot. Ask each group to work cooperatively to create a chart, table, or graph to organize their scores. Have the students prepare a presentation for the class, making sure they give the prediction, actual number of shots made out of 20, and each student's probability of making the next shot.

Activity 4: It's Theoretical! (GLE: 38)

Materials List: coins, graphing calculator (optional), paper, pencil, dice, cups, spinners

Introduce the activity by asking students, "What is the probability of getting a head when we toss a single coin? The probability of getting a tail?" After giving time for students to provide an answer, ask how they determined the answer. Determine that students understand that the probability of getting a head is $\frac{1}{2}$, and the probability of getting a tail is $\frac{1}{2}$, which is called the theoretical probability. Theoretical probability is determined mathematically by comparing the number of possible favorable outcomes (what you want

to happen) to the total number of possible outcomes for a particular event. In this instance, there are two possible outcomes because a coin has two sides. Only one side has a head, so the number of possible favorable outcomes is only one.

Have each student toss a coin 10 times, keeping track of the results. Ask students if they got the same number of heads as tails. Did each result (heads and/or tails) occur the same number of times? What is the probability of getting a head according to the data? This is called *experimental probability*. Combine all the data from the class members and then recalculate the number of heads versus the number of tails. Have the students compare their individual results of tossing the coin 10 times with the class results. Discuss as a class. (If students have a graphing calculator available, the coin toss could be done 10 times, then 100 times to make a comparison of the results.) Lead students to understand that the more times the experiment is carried out, the closer the results of the experimental probability get to the theoretical probability.

Create activity centers for students to work in groups.

- Cups with dice (e.g., 1 cup with 1 die; 1 cup with 2 dice, and so on)
- Spinning arrows on cardboard (e.g., 1 with only 2 marked off sections; 1 with 4 different sections, and so on)

Have students determine the theoretical probability of certain situations, and then compare with experimental probability based on results of their experiments. For example, have students determine the theoretical probability of rolling a 6 when rolling one die and the probability of getting a sum of 4 when rolling a pair of dice. Then have students perform the experiments and compare the theoretical and experimental probabilities.

Activity 5: How Do the Chips Fall? (GLEs: 37, 38)

Materials List: marbles or chips (blue, red, and green), How Do the Chips Fall? BLM, pencils, paper bags

Before class, prepare a paper bag with 5 blue marbles/chips, 3 red marbles/chips, and 2 green marbles/chips for each pair of students.

Have students work in pairs for this experiment. Give each pair one of the prepared bags and one copy of How Do the Chips Fall? BLM. Have the students look in their bag, find the theoretical probability of each color of marble/chip, and record it on the chart found on How Do the Chips Fall? BLM.

Each group will now conduct an experiment and record the results on the How Do the Chips Fall? BLM. Student 1 draws a marble from the bag, Student 2 tallies the color, Student 1 returns the marble to the bag. Have students repeat the process for a total of 25 draws.

Using the results from the experiment, have students complete the How Do the Chips Fall? BLM, computing the frequency and experimental probability, then compare the experimental and theoretical probabilities. As a class, discuss their comparisons. Give each pair a second copy of How Do the Chips Fall? BLM, and instruct students to switch roles and repeat the experiment with Student 2 drawing and returning the marbles 25 times and Student 1 acting as the recorder.

Combine results for Student 1 and Student 2. How do the experimental and theoretical probabilities compare now? How do the individual experimental probability and the combined experimental probability compare? Discuss. Find the probabilities using the results obtained by the entire class and compare them to the theoretical probabilities.

Have students use a modified *GISTing* ([view literacy strategy descriptions](#)) strategy to write an accurate 4-5 sentence summary of experimental and theoretical probabilities. Students may refer to the information they collected from How Do the Chips Fall? BLM. This is a modified *GIST* because there is no text for students to paraphrase. Students should be able to summarize their findings very succinctly.

Sample *GIST*: From a bag containing 5 blue chips, 3 red chips, and 2 green chips, the theoretical probability of picking a blue chip is 5 out of 10 or 50%. The experimental probability that I will pick a blue chip is 13 out of 25 since I picked a chip out of the bag 25 times and 13 of those times I picked a blue chip.

Activity 6: Birthdays (GLEs: 32, 37)

Materials List: pencil, paper

Have the students collect data from the class to find the month each person was born. Have students organize the data in a line plot. Instruct students to analyze the data identifying any clusters, gaps, outliers, or patterns.

Ask the students to find different probabilities from the data such as, “What is the probability that a student was born in July? After April?” Add your month to the data, and have the students describe how the probabilities of the questions changed. Ask what would happen to the probabilities if data from other classes were added to the line plot.

Activity 7: Probability with *Jumanji* (GLEs: 36, 37, 38)

Materials List: *Jumanji*, paper, pencil, dice, *Jumanji* BLM

Read the book *Jumanji* to the students. While reading the book, stop at different points in the book to ask mathematical questions. For example, on page 6 (where the two children are sitting at the table with the game board), say, “To play the game of *Jumanji*, the children rolled two dice and found the sum. If there are 48 spaces on the game board, what is the least number of plays it would take one person to win the game? Explain.”

Continue reading the book and asking questions periodically.

After reading the entire book, give each pair of students a pair of dice and the Jumanji BLM. Instruct the students to create a list of the different ways the dice could land. How many ways are there? (36) Point out that this is an application of the fundamental counting principle. From their lists, have students find the theoretical probability of rolling the sums 2 through 12. Students should roll the pair of die 12 times and record the sum of the roll each time then find the experimental probability of getting each sum. Have the students compare experimental and theoretical probability from the roll of their dice.

Sample Assessments

General Assessments

- Determine student understanding as the student engages in the various activities.
- Whenever possible, create extensions to an activity by increasing the difficulty or by asking “what if” questions.
- The student will be encouraged to create his/her own questions to evaluate his/her understanding of theoretical and experimental probabilities.
- Use checklists/rubrics to judge correctness and accuracy of calculations and presentations of counting principles.
- The student will provide a portfolio item to show understanding of one of the concepts in the unit, such as these:
 - The student will create and write a situation that illustrates the counting principle and explain the work.

The student will complete math *learning log* ([view literacy strategy descriptions](#)) entries using such topics as these:

- Compare and contrast *theoretical* and *experimental* probability
- Explain whether or not order matters when finding arrangements for specific situations

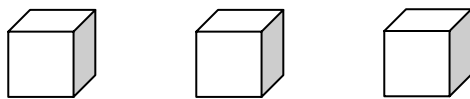
Activity-Specific Assessments

- Activity 4: The student will solve the following problem:
When tossing a coin, there are 2 possible outcomes heads or tails. Andy flipped a coin 3 times. Every time the coin came up heads. Andy is going to flip the coin again. What is the chance of getting a head the 4th time? Explain your answer.

(Solution: There is an equal amount of getting heads or tails (1 out of 2 chances), it doesn't matter what he has flipped before.)

- Activity 4: The student will solve the following problem:
You have a fair spinner divided into ten equal portions numbered 1 through 10.
 1. What is the probability of spinning an even number? (*Solution: 5 out of 10 or $\frac{1}{2}$*)
 2. What is the probability of spinning a 3? (*Solution: 1 out of 10 or $1/10$*)
 3. What is the probability of spinning an even number followed by an odd number? (*Solution: 1 out of 4 or $\frac{1}{4}$*)
 4. What is the probability of spinning a number greater than 8? (*Solution: 2 out of 10 or $1/5$*)
- Activities 4, 5 and 7: The student will provide a correct solution to the problem below.

Dora has three different colored number cubes with sides numbered 1 through 6.



List all possible ways that a sum of 10 could be made when all three number cubes are tossed at the same time.

(*Solution: R3, B2, G5; R5, B3, G2; R3, B5, G2; R5, B2, G3; R1, B4, G5; R1, B5, G4; R4, B1, G5; R4, B5, G1; R5, B1, G4; R5, B4, G1 10 different tosses*)

**Grade 7
Mathematics
Unit 8: Introduction to Algebraic Problem Solving**

Time Frame: Approximately four weeks



Unit Description

This unit connects equation solving to representation of problem situations from an algebraic standpoint. While these problems may often be solved numerically or through mental math methods, this unit develops algorithmic methods of algebra and the related techniques that are used to solve problems involving growth, change, variation, and numerical relationships.

Student Understandings

Students should be able to solve one- and two-step equations and also solve and represent solutions to inequalities on a number line. They should be able to represent growth (linear and exponential), or alternatively (arithmetic or geometric), in equation or expression form. Students also use function machines' input-output tables and their graphs in making similar analyses of data and relationships.

Guiding Questions

1. Can students solve problems involving one- and two-step contexts with fractions, decimals, and integers?
2. Can students represent such problems in equations and inequalities and solve the respective forms?
3. Can students recognize and solve problems involving linear or exponential growth, distinguishing between them and their representations?
4. Can students match real-life situations with their algebraic models and write equations/inequalities as needed?

Unit 8 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
3.	Solve order of operations problems involving grouping symbols and multiple operations (N-4-M)

GLE #	GLE Text and Benchmarks
7.	Select and discuss appropriate operations and solve single- and multi-step, real-life problems involving positive fractions, percents, mixed numbers, decimals, and positive and negative integers (N-5-M) (N-3-M) (N-4-M)
Algebra	
14.	Write a real-life meaning of a simple algebraic equation or inequality, and vice versa (A-1-M) (A-5-M)
16.	Solve one- and two-step equations and inequalities (with one variable) in multiple ways (A-2-M)
17.	Graph solutions sets of one-step equations and inequalities as points, or open and closed rays on a number line (e.g., $x = 5$, $x < 5$, $x \leq 5$, $x > 5$, $x \geq 5$) (A-2-M)
18.	Describe linear, multiplicative, or changing growth relationships (e.g., 1, 3, 6, 10, 15, 21, ...) verbally and algebraically (A-3-M) (A-4-M) (P-1-M)
19.	Use <i>function machines</i> to determine and describe the rule that generates outputs from given inputs (A-4-M) (P-3-M)
Geometry	
25.	Draw the results of reflections and translations of geometric shapes on a coordinate grid (G-3-M)
27.	Model and explain the relationship between perimeter and area (how scale change in a linear dimension affects perimeter and area) and between circumference and area of a circle (G-5-M)
29.	Plot points on a coordinate grid in all 4 quadrants and locate the coordinates of a missing vertex in a parallelogram (G-6-M) (A-5-M)
Data Analysis, Probability, and Discrete Math	
33.	Analyze discrete and continuous data in real-life applications (D-2-M) (D-6-M)
Patterns, Relations, and Functions	
40.	Analyze and verbally describe real-life additive and multiplicative patterns involving fractions and integers (P-1-M) (P-4-M)

Sample Activities

Activity 1: Solve Order of Operations Problems Using Symbols and Multiple Operations (GLE: 3)

Materials List: box with numbers or dice, newsprint, color markers, white boards and dry erase markers or index cards

Review the order of operations and steps. Give students the following set of problems or a similar set to review the order of operations. After students have worked the set, allow them to compare their answers with another student, then have pairs form groups of 4 to compare answers with another pair. Have each group give the answer to one or more problems and explain how it arrived at its answer.

Problem Set:

$$6 + 3 \times 2^2 = 18 \qquad 4(2 + 3^2) = 44 \qquad \frac{27-7}{2} + 11 = 21$$

$$\left(\frac{36+14}{5}\right)^2 = 100 \qquad 50 + 10 \div 2 - 3 \times 5 = 40$$

$$\frac{4 + 3 \times 4}{5 - 3} = 8 \qquad \frac{100}{9 + 4^2} = 4 \qquad (10 - 2.6 \div 2 - 5.7)^3 = 27$$

Have students, working in groups of four, select 4 numbers from a box or from rolling a die. Students are to use the four numbers and any order of operations to write an expression, write the expression on a blank piece of newsprint, and then calculate the answer on another sheet of paper. The expression will be used to play a game called *Challenge*. When finished, one group will present its expression to the class. Remaining groups will simplify the expression. The presenting team will earn one point for each group that it “stumps.” To assist in determining which teams have correct answers, give each team a small white board and dry eraser marker on which to show work and write an answer. Another alternative is to provide each group with one index card for each problem to be worked. Set a two-three minute time limit for each problem, and have students show their solutions after time is called.

Activity 2: Game of 24 (GLE: 3)

Materials List: fair spinners, number cubes or graphing calculators, pencil, paper, optional-decks of playing cards

Play Game of 24 in groups of four students. Provide each group with a fair spinner, a number cube, or a graphing calculator to generate random numbers. Ask Student 1 to generate 4 random numbers for the group to use in round 1. Have each student in the group use the 4 numbers to create an order of operations problem that has a solution of 24. Allow students to add, subtract, multiply, divide, use exponents, and/or parentheses with each of the 4 numbers being used only once in the problem.

When a player thinks he has a correct solution, instruct him/her to declare, “I got it first.” Allow other players to continue working and declare “I got it second (third or fourth)” as they finish the task. Indicate that the winner of the round is the *first* person to declare “I got it” and has the correct answer. Have the group check the solution for the first person. If the solution is correct, award a point to the student. If it is incorrect, check the next person who said “I got it,” and continue the process to determine if there is a winner for the round.

To increase the difficulty, have students use a deck of cards to randomly pick the four numbers. Red cards are negative numbers; black cards are positive numbers. Allow an

ace to count as either 1/-1 or 11/-11 depending on the color. Face cards (jack, queen, king) have a value of 10 or -10, again depending on the color.

For subsequent rounds, give each of the other group members a chance to generate the random numbers. Award a small prize to the group member who has the most points at the end of the game.

Activity 3: Solving Equations (GLEs: 14, 16)

Materials List: tokens, pencil, paper

Review how to solve one and two step equations. Be sure to include negative numbers. Have students work several equations of each kind, and explain how they arrived at their answer. Here are some examples of equations that could be used to review.

$$2a = 24; a = 12 \qquad 4 + y = -16; y = -20 \qquad \frac{x}{-2} = 22; x = -11$$

$$4d + 2 = 14; d = 3 \qquad \frac{-8s}{4} = 2; s = -1 \qquad -5n + (-3) = -23; n = 4$$

Divide the class into groups of three and give each group 50 tokens each labeled with the same letter. Make sure to use a different letter for each group when labeling the tokens (e.g., if there are 9 groups, use A, B, C, D, E, F, G, H, I). Ask each group of students to choose a value (between 1 and 10) for their tokens. Go to each group and record the value for each labeled token (e.g., 'A' tokens = 2, 'B' tokens = 5) so that members of a group know only the value of their own tokens. Have each group trade some of their tokens with *one* other group. Instruct each group to combine a few of their tokens (with known value) with some of the traded tokens (of unknown value) to form a new set containing tokens of two letters. Walk around the room and tell each group the value of its newly created set of tokens. Ask students to write equations representing the value of their newly created set of tokens (e.g., $3A + 2B = 16$) and to use the known value to determine the value of the unknown tokens (e.g., If $A = 2$, find B.).

Activity 4: Greater Than, Less Than (GLEs: 14, 16, 17, 33)

Materials List: Dogs BLM, pencil, magazines, Internet (optional), scissors, glue or tape, chart paper, color markers

Demonstrate the concept of greater than and less than using the chart provided on Dogs BLM or make a similar chart from information found at <http://www.enchantedlearning.com/subjects/mammals/dog/index.shtml>. Tell students that a certain Pekingese weighs 11 pounds, and ask them to graph the dog's weight on a number line. The graph should be a solid dot/closed circle on a number line at 11. Next tell students that an average Pekingese weighs more than 7 pounds, and have them graph this weight on a number line. The graph should be an open circle at 7 with the number

line shaded to the right of 7. Now ask students what the graph would look like if the average weight of a Pekingese were at least 7 pounds, and ask them to graph this weight; $w \geq 7$. The graph should be a closed circle at 7 with the number line shaded to the right of 7. Have students describe the difference between more than 7 pounds and at least 7 pounds. *More than 7 pounds means the weight cannot be 7 pounds, but can be any weight more than 7 pounds; there can be more than one weight. At least 7 pounds means the dog could weight exactly 7 pounds or any weight more than 7 pounds.* Make sure students can also describe the difference between less than and less than or equal to.

Ask students to write an inequality to represent “the weight of a dog plus 10 pounds is no more than 27 pounds” or $w + 10 \leq 27$. Have students explain how to find the value of w . Show several additional examples of inequalities and how to solve them.

Pair students and give each pair a copy of Dogs BLM. Using Dogs BLM, have each pair of students choose 10 different dogs the dog lover could take on the airplane. Next have students determine the different combinations of dogs that a person can take on the airplane. (e.g., 10 Maltese, two Cocker Spaniels or one German Shepherd if the limit is 100 pounds).

Have students use the chart and pictures from magazines or Internet sites to design their own situations with inequalities. Using chart paper, have students create a situation, write an inequality for the situation, and solve it, showing the solution on a number line. Allow students to present this information to the class. Ask, “Would these graphs represent continuous or discrete data?” Have students explain.

Activity 5: Interpret Function Machines (GLEs: 7, 19)

Materials List: list of countries and exchange rates, Internet access, Exchange Rates BLM, pencil

Tell students that they are taking a trip to many countries and need a function machine that will change dollars to pesos, euros, yen, and so on. Tell groups of students which country they are to visit, and have students find exchange rates and create a function machine using Exchange Rates BLM. Allow them to use <http://currency.buy-get.com/> for current exchange rates, or provide the students with the exchange rates. Have each group complete the chart on Exchange Rates BLM to show conversions of standard denominations of bills.

Instruct students to assume the role of *professor know-it-all* ([view literacy strategy descriptions](#)) when ready to present their chart and function machine. Other students should listen closely for logic and reasonableness. The students presenting should be able to defend their answers and reasoning.

Activity 6: Graphing Ordered Pairs (GLEs: 25, 27, 29)

Materials List: 1 cm graph paper, pencils, ruler, scissors

Have the students plot the points $A(1,3)$, $B(5,3)$, $C(5,5)$, $D(13,5)$

$E(13,7)$, $F(10,7)$, $G(10,11)$, $H(1,11)$ on a coordinate grid. Have the students label and connect each point as it is plotted to form a polygon. Ask students to:

- find the area and perimeter of the polygon
- reflect the polygon over the x -axis then provide the new coordinates
- translate the *original* figure to the right 2 and down 1, then provide the new coordinates

Have the students cut each original ordered pair in $\frac{1}{2}$ and then plot the new shape using these ordered pairs. Introduce the terms *dilation*, *reduction*, and *enlargement*. Ask students to find the area and perimeter of this figure and the area and perimeter of the original polygon. Discuss the students' findings. Ask if the process results in a reduction or enlargement of the original figure.

Have students create their own polygons on a coordinate grid, and then enlarge/reduce the originals by doubling and then halving the coordinates. Have students compare the areas and perimeters of the three figures.

Instruct the students to write an accurate summary or *GISTing* ([view literacy strategy descriptions](#)) for creating dilations. This is a modification of the GISTing process because there is no text for students to paraphrase; however, students should be able to express how to create dilations (reductions and enlargements) in their own words, using math symbols and diagrams.

Sample GIST: "When a figure is reduced, the scale factor is less than one. When a figure is enlarged, the scale factor is greater than one. When the scale factor is one, the dilation is an exact copy. To find the new coordinates, multiply each coordinate by the scale factor.

Activity 7: Find the Pattern (GLEs: 14, 18, 19, 40)

Materials List: pencil, paper

Present the following scenario to the students: Carolyn is renting a car for one week from Friendly Auto Rentals. Friendly charges \$225 for a weekly rental plus \$0.25 per mile. Have students create a chart in increments of 10s or 20s showing the total cost for the rental if Carolyn drives up to 100 miles. Ask students to find the rule for the total cost, c , of renting the car when driving m miles. Lead students in a discussion about students' findings.

At Save More Rentals, the weekly rental is \$150 plus \$0.40 per mile. Have students create a similar chart showing the total cost for the rental if Carolyn drives up to 100 miles. Ask how this information changes the equation (rule).

Rent Here Rentals charges \$0.80 per mile and no weekly fee. Have students create a function machine in the form of a chart showing the total cost for the rental if Carolyn drives up to 100 miles.

Have students determine which plan would be least expensive if Carolyn wants to rent the car for one week and drive 400 miles.

Additional patterns showing various growth relationships should be explored. Sources for the patterns could include Internet sites, newspaper lists, almanac data, and so on.

Solutions:

Friendly Auto Rentals $225 + .25m = c$

Miles (m)	10	20	30	40	50	60	70	80	90	100	400
Cost (c)	\$227.50	\$230	\$232.50	\$235	\$237.50	\$240	\$242.50	\$245	\$247.50	\$250	\$325

Save More Rentals $150 + .40m = c$

Miles (m)	10	20	30	40	50	60	70	80	90	100	400
Cost (c)	\$154	\$158	\$162	\$166	\$170	\$174	\$178	\$182	\$186	\$190	\$310

Rent Here Rentals $100 + .80m = c$

Miles (m)	10	20	30	40	50	60	70	80	90	100	400
Cost (c)	\$108	\$116	\$124	\$132	\$140	\$148	\$156	\$164	\$172	\$180	\$420

If Carolyn is going only 100 miles, it would be best for her to rent from Save More Rentals. If she is going 400 miles, it would be best for her to rent from Rent Here Rentals.

Activity 8: Patterns (GLEs: 14, 18, 19, 40)

Materials List: pencil, paper, 1 cm graph paper

A pattern of squares is shown. Ask students to

- Sketch the fourth and fifth figure in this pattern.
- Determine how many squares would be in the 10th figure.
- Make a function machine in the form of a table comparing the figure number to the number of squares, and write an expression for the number of squares in the n^{th} figure.

- d. Determine how many squares would be in the 100th figure.
- e. Graph the data on 1 cm graph paper (figure number vs. number of squares).

figure 1

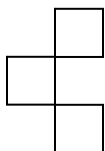


figure 2

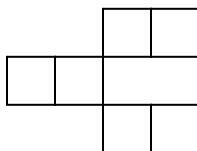
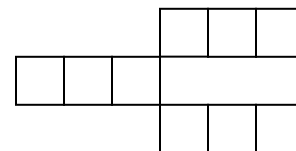


figure 3



Solutions: to b, c, d found in table below

Figure Number	Number of squares
1	3
2	6
3	9
4	12
5	15

Figure Number	Number of squares
10	30
100	300
n	$3n$

Activity 9: Campsites (GLEs: 14, 18, 29, 40)

Materials List: pencil, math *learning log*, 1 cm grid paper, glue or tape

Have students respond to the following prompt in their math *learning log* ([view literacy strategy descriptions](#)). Students will need to glue or tape 1 cm grid paper in their math *learning log*.

The following table shows the fees charged for campsites at one of the campgrounds at Sam Houston State Park.

C = Number of campsites	1	2	3	4	5	6	7	8
F = Total campground fees	\$12.50	25.00	37.50	50.00	62.50	75.00	87.50	100.00

- a. Make a graph of this data on a coordinate grid. Would it make sense to use negative numbers on this graph? Why or why not?
- b. What will you label the x -axis and the y -axis? Why did you choose this?
- c. Would it make sense to connect the points on the graph? Why or why not?

- Activity 3: The student will correctly work the following problem:
Mrs. Smith is investing money in stocks to help pay for her child's college education. She buys one share of NIKE stock at \$16. She also purchases five shares of Disney stock at \$35 each. Write an expression that represents the amount of money Mrs. Smith spends for stock purchases. Find the total amount of money Mrs. Smith invests.
Solution: $16 + 35 \times 5 =$ Mrs. Smith invests \$191 in stocks
- Activity 6: The student will correctly work the following problem and justify the answer:
On a coordinate grid, the points $(2,1)$, $(2,2)$, and $(-5,-2)$ are three corners of a rectangle. Without plotting the points, determine the coordinates of the fourth corner. Explain how you determined this point.