Most of the math symbols in this document were made with Math Type® software. Specific fonts must be installed on the user’s computer for the symbols to be read. It is best to use the pdf format of a document if a printed copy is needed.

To copy and paste from the Word document, download and install the Math Type® for Windows Font from http://www.dessci.com/en/dl/fonts/default.asp on each computer on which the document will be used.
1.1 *Function of x* – define function, how to identify equations as functions of \( x \), how to identify graphs as functions of \( x \), how to determine if ordered pairs are functions of \( x \), how to explain the meaning of \( f(x) \) (e.g., If \( f(x) = 3x^2 - 4 \), find \( f(3) \) and how to explain the process used in terms of a function machine.)

1.2 *Four Ways to Write Solution Sets* – explain/define roster, interval notation using \( \cup \), number line, set notation using “and” or “or”.

1.3 *Absolute Value Equations* and *Inequalities as Solution Sets* – write solutions in terms of “distance,” change absolute value notation to other notations and vice versa (e.g., write \(|x| < 4, |x - 5| \leq 6, |x| \geq 9\) as number lines, as words in terms of distance, as intervals, and in set notation; write \( : [-8, 8] \), \((-4, 6)\) in absolute value notation.).

1.4 *Domain and Range* – write the definitions, give two possible restrictions on domains based on denominators and radicands, determine the domain and range from ordered pairs, graphs, equations, and inputs and outputs of the function machine; define abscissa, ordinate, independent variable, and dependent variables.

1.5 *Slope of a Line* – define slope, describe lines with positive, negative, zero and no slope, state the slopes of perpendicular lines and parallel lines.

1.6 *Equations of Lines* – write equations of lines in slope-intercept, point-slope, and standard forms, and describe the process for finding the slope and \( y \)-intercept for each form.

1.7 *Distance between Two Points and Midpoint of a Segment* – write and explain the formula for each.

1.8 *Piecewise Linear Functions* – define and explain how to find domain and range for these functions. (e.g., Graph and find the domain and range of \( f(x) = \begin{cases} 2x + 1 & \text{if } x > -3 \\ -x - 5 & \text{if } x \leq -3 \end{cases} \)

1.9 *Absolute Value Function* – define \( y = |x| \) as a piecewise function and demonstrate an understanding of the relationships between the graphs of \( y = |x| \) and \( y = a|x - h| + k \) (i.e., domains and ranges, the effects of changing \( a, h, \) and \( k \)). Write \( y = 2|x - 3| + 5 \) as a piecewise function, explain the steps for changing the absolute value equation to a piecewise function, and determine what part of the function affects the domain restrictions.

1.10 *Step Functions* and *Greatest Integer Function* – define each and relate to the piecewise function. Graph the functions and find the domains and ranges. Work and explain how to work the following examples: (1) Solve for \( x \): \( \left[ \frac{1}{2} x \right] = 7 \). (2) If \( f(x) = \left[ 2x - 5 \right] + 3 \), find \( f(0.6) \) and \( f(10.2) \).

1.11 *Composite Functions* – define, find the rules of \( f(g(x)) \) and \( g(f(x)) \) using the example, \( f(x) = 3x + 5 \) and \( g(x) = x^2 \), interpret the meaning of \( f \circ g \), explain composite functions in terms of a function machine, explain how to find the domain of composite functions, and how to graph composite functions in the graphing calculator.

1.12 *Inverse Functions* – define, write proper notation, find compositions, use symmetry to find the inverse of a set of ordered pairs or an equation, determine how to tell if the inverse relation of a set of ordered pairs is a function, explain how to tell if the inverse of an equation is a function, and explain how to tell if the inverse of a graph is a function.
Determine if each of the following is a function of $x$. Explain both yes and no answers.

(1) the set of ordered pairs 
   $\{(x, y): (1, 2), (3, 5), (3, 6), (7, 5), (8, 2)\}$
(2) $\{(x, y): (1,1), (2,4), (3,9), (-1,1), (-2,4), (-3,9)\}$
(3) the relationship “$x$ is a student of $y$”
(4) the relationship “$x$ is the biological daughter of mother $y$”
(5) the equation $2x + 3y = 6$
(6) the equation $x + y^2 = 9$
(7) the equation $y = x^2 + 4$
(8)
Domain & Range in Real World Applications

Complete the following chart:

<table>
<thead>
<tr>
<th>Application</th>
<th>Function Notation</th>
<th>Independent Variable</th>
<th>Allowable Values of the Independent Variable</th>
<th>Dependent Variable</th>
<th>Resulting Values of the Dependent Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) The area of a circle depends on its radius</td>
<td>$A(r) = r^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) The length of the box is twice the width, thus depends on the width</td>
<td>$l(w) = 2w$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3) The state tax on food is 5%, and the amount of tax you pay depends on the cost of the food bought</td>
<td>$t(c) = 0.05c$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4) $d$ depends on $s$ in a set of ordered pairs, ${(s, d): (1,1), (2, 4), (3, 9), (-1, 1), (-2, 4), (-3, 9)}$</td>
<td>$d(s) = s^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Define **domain**: ___________________________________________________________________

Specify the **domains** of the above functions in **interval notation** and explain why they are restricted:

(1) Domain: ___________ Why restricted? ___________________________________________________________________
(2) Domain: ___________ Why restricted? ___________________________________________________________________
(3) Domain: ___________ Why restricted? ___________________________________________________________________
(4) Domain: ___________ Why restricted? ___________________________________________________________________

- Define **range**: ___________________________________________________________________

Specify the **ranges** of the above functions in **interval notation** and explain why they are restricted:

(1) Range: ___________ Why restricted? ___________________________________________________________________
(2) Range: ___________ Why restricted? ___________________________________________________________________
(3) Range: ___________ Why restricted? ___________________________________________________________________
(4) Range: ___________ Why restricted? ___________________________________________________________________
**Unit 1, Activity 3, Domain & Range Discovery Worksheet**

**Domain & Range from Graphs**

In the following graphs, what is the independent variable? _____ the dependent variable? _____

State the domain and range of the following graphs using interval notation. Assume the graphs continue to infinity as the picture leaves the screen.

**Reviewing Absolute Value Notation:** State the domain and range of #6 and #9 above in absolute value notation:

(11) Domain of #6: ___________________  
     Range: ___________________  

(12) Domain of #9: ___________________  
     Range: ___________________
**Domain & Range from Algebraic Equations**

Consider the following functions.

- Decide if there are any values of $x$ that are not allowed therefore creating a restricted domain. State the domain of each function in set notation and why it is restricted.
- Then consider if this restricted domain results in a restricted range. State the range of each function in set notation and why it is restricted.

<table>
<thead>
<tr>
<th>Function</th>
<th>Domain and Why Restricted</th>
<th>Range and Why Restricted</th>
</tr>
</thead>
<tbody>
<tr>
<td>(13) $f(x) = 3x + 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(14) $f(x) = \frac{1}{x}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(15) $g(x) = \sqrt{x}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(16) $f(x) = \frac{1}{2x - 6}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(17) $g(x) = -\sqrt{x - 2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(18) Explain two types of domain restrictions in the real number system demonstrated by the examples above:

I. __________________________________________

II. __________________________________________

**Combinations of Functions** When a third function is created from the combination of two functions, the domain of the combination must include the domains of the original function further restricted by the new combination function.

- $f(x) = \sqrt{x - 2}$  What is the domain of $f(x)$? ______________
- $g(x) = \frac{1}{x - 3}$  What is the domain of $g(x)$? ______________

Find the equation for the following combinations and determine the new domain in set notation:

(19) $(f + g)(x) =$ ________________  Domain: ________________

(20) $(fg)(x) =$ ________________  Domain: ________________

(21) $\frac{g}{f}(x) =$ ________________  Domain: ________________
**Unit 1, Activity 3, Domain & Range Discovery Worksheet with Answers**

Complete the following chart:

<table>
<thead>
<tr>
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<tr>
<td>(1) The area of a circle depends on its radius</td>
<td>( A(r) = r^2 )</td>
<td>( r )</td>
<td>( r \geq 0 )</td>
<td>( A )</td>
<td>( A \geq 0 )</td>
</tr>
<tr>
<td>(2) The length of the box is twice the width thus depends on the width</td>
<td>( l(w) = 2w )</td>
<td>( w )</td>
<td>( w \geq 0 )</td>
<td>( l )</td>
<td>( l \geq 0 )</td>
</tr>
<tr>
<td>(3) The state tax on food is 5%, and the amount of tax you pay depends on the cost of the food bought</td>
<td>( t(c) = 0.05c )</td>
<td>( c )</td>
<td>( c \geq 0 )</td>
<td>( t )</td>
<td>( t \geq 0 )</td>
</tr>
<tr>
<td>(4) ( d ) depends on ( s ) in a set of ordered pairs, {((s, d): (1,1), (2, 4), (3, 9), (-1, 1), (-2, 4), (-3, 9)})</td>
<td>( d(s) = s^2 )</td>
<td>( s )</td>
<td>{(1, 2, 3, -1, -2, -3)}</td>
<td>( d )</td>
<td>{(1, 4, 9)}</td>
</tr>
</tbody>
</table>

- Define **domain**: the allowable values of the independent variable.
- Define **range**: the resulting values of the dependent variable.

Specify the domains of the above functions in interval notation and explain why they are restricted:

1. Domain: \([0, \infty)\) Why restricted? **because all radii are \(> 0\)**
2. Domain: \((0, \infty)\) Why restricted? **because all widths \(> 0\)**
3. Domain: \([0, \infty)\) Why restricted? **because all costs \(> 0\)**
4. Domain: \(\{1, 2, 3, -1, -2, -3\}\) Why restricted? **cannot write as interval only roster first terms**

Specify the ranges of the above functions in interval notation and explain why they are restricted:

1. Range: \([0, \infty)\) Why restricted? **because all areas \(> 0\)**
2. Range: \((0, \infty)\) Why restricted? **because all lengths \(> 0\)**
3. Range: \([0, \infty)\) Why restricted? **because all taxes \(> 0\)**
4. Range: \(\{1, 4, 9\}\) Why restricted? **cannot write as interval only roster second terms**
Domain & Range from Graphs

In the following graphs, what is the independent variable? \( x \)  the dependent variable? \( y \)

State the domain and range of the following graphs using interval notation. Assume the graphs continue to infinity as the picture leaves the screen.

(5) Domain: \((-\infty,\infty)\)  
Range: \([4,\infty)\)

(8) Domain: \((-\infty,\infty)\)  
Range: \([-8,\infty)\)

(6) Domain: \((-\infty,0)\cup(0,\infty)\)  
Range: \((-\infty,0)\cup(0,\infty)\)

(9) Domain: \([-3,3]\)  
Range: \([0,3]\)

(7) Domain: \([0,\infty)\)  
Range: \((-\infty,5]\)

(10) Domain: \((-\infty,-1]\cup[2,\infty)\)  
Range: \([-3,\infty)\)

Reviewing Absolute Value Notation: State the domain and range of #6 and #9 above in absolute value notation:

(11) Domain of #6: \(|x| > 0\)  
Range: \(|y| > 0\)

(12) Domain of #9: \(|x| \leq 3\)  
Range: \(|y - 1.5| \leq 1.5\)
**Domain & Range from Algebraic Equations**

Consider the following functions.

- Decide if there are any values of $x$ that are not allowed therefore creating a restricted domain. State the domain of each function in set notation and why it is restricted.
- Then consider if this restricted domain results in a restricted range. State the range of each function in set notation and why it is restricted.

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<th>Range and Why Restricted</th>
</tr>
</thead>
<tbody>
<tr>
<td>(13) $f(x) = 3x + 1$</td>
<td>${x : x \in \text{Reals}} \text{ no restrictions}$</td>
<td>${y : y \in \text{Reals}} \text{ no restrictions}$</td>
</tr>
<tr>
<td>(14) $f(x) = \frac{1}{x}$</td>
<td>${x : x \neq 0} \text{ Division by zero is undefined.}$</td>
<td>${y : y \neq 0} \text{ Because the numerator is a constant, } y \text{ will never result in the value 0.}$</td>
</tr>
<tr>
<td>(15) $g(x) = \sqrt{x}$</td>
<td>${x : x \geq 0} \text{ You cannot take a square root of a negative number}$</td>
<td>${y : y \geq 0}, \text{ A radical is always the principal square root therefore always positive or zero.}$</td>
</tr>
<tr>
<td>(16) $f(x) = \frac{1}{2x - 6}$</td>
<td>${x : x \neq 3} \text{ Division by zero is undefined}$</td>
<td>${y : y \neq 0} \text{ Because the numerator is a constant, } y \text{ will never result in the value 0.}$</td>
</tr>
<tr>
<td>(17) $g(x) = -\sqrt{x-2}$</td>
<td>${x : x \geq 2}, \text{ You cannot take a square root of a negative number.}$</td>
<td>${y : y \leq 0}, \text{ A radical is always the principal square root therefore always positive or zero. The negative in front of the radical makes it always negative or zero.}$</td>
</tr>
</tbody>
</table>

(18) Explain two types of domain restrictions in the real number system demonstrated by the examples above:

I. **Division by zero is undefined,**

II. **The value under the square root (or any even root) must be > 0.**

**Combinations of Functions**

When a third function is created from the combination of two functions, the domain of the combination must include the domains of the original functions further restricted by the new combination function.

- $f(x) = \sqrt{x-2}$ What is the domain of $f(x)?$ $x \geq 2$
- $g(x) = \frac{1}{x-3}$ What is the domain of $g(x)?$ $x \neq 3$

Find the equation for the following combinations and determine the new domain in set notation:

(19) $(f + g)(x) = (f + g)(x) = \sqrt{x-2} + \frac{1}{x-3}$ Domain: $\{x \geq 2, x \neq 3\}$

(20) $(fg)(x) = (fg)(x) = \frac{\sqrt{x-2}}{x-3}$ Domain: $\{x \geq 2, x \neq 3\}$

(21) $\frac{g}{f}(x) = \frac{g}{f}(x) = \frac{1}{(x-3)\sqrt{x-2}}$ Domain: $\{x > 2, x \neq 3\}$
**Unit 1, Activity 5, Linear Equation Terminology**

Name_________________________________________ Date____________________

**Vocabulary Self–Awareness Chart**

Complete the following with a partner.
- Rate your understanding of each concept with either a “+” (understand well), “✓” (limited understanding or unsure), or a “−” (don’t know)
- Write the formula or description

<table>
<thead>
<tr>
<th>Mathematical Terms</th>
<th>+</th>
<th>✓</th>
<th>−</th>
<th>Formula or description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 slope of a line</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 slope of horizontal line</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 equation of a horizontal line</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 slope of a line that starts in Quadrant III and ends in Quadrant I</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 slope of a line that starts in Quadrant II and ends in Quadrant IV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 slope of a vertical line</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 equation of a vertical line</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 slopes of parallel lines</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 slopes of perpendicular lines</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 point-slope form of equation of line</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11 $y$-intercept form of equation of line</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 standard form of equation of line</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13 distance formula</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14 midpoint formula</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Sample Problems
Create a sample problem for each concept and solve it: *(The first one has been created for you as an example, but you still have to solve it.)*

<p>| | | |</p>
<table>
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<tr>
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<th></th>
</tr>
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<tbody>
<tr>
<td>1) slope of a line</td>
<td>8) slopes of parallel lines</td>
<td></td>
</tr>
<tr>
<td><em>Find the slope of the line between the two points (−2, 6) and (9, 4)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) slope of horizontal line</td>
<td>9) slopes of perpendicular lines</td>
<td></td>
</tr>
<tr>
<td>3) equation of a horizontal line</td>
<td>10) point-slope form of equation of line</td>
<td></td>
</tr>
<tr>
<td>4) slope of a line that starts in Quadrant III and ends in Quadrant I</td>
<td>11) $y$-intercept form of equation of line</td>
<td></td>
</tr>
<tr>
<td>5) slope of a line that starts in Quadrant II and ends in Quadrant IV</td>
<td>12) standard form of equation of line</td>
<td></td>
</tr>
<tr>
<td>6) slope of a vertical line</td>
<td>13) distance formula</td>
<td></td>
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<tbody>
<tr>
<td>1 slope of a line</td>
<td></td>
<td>$m = \frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{\Delta y}{\Delta x}$</td>
</tr>
<tr>
<td>2 slope of horizontal line</td>
<td></td>
<td>$m = 0$</td>
</tr>
<tr>
<td>3 equation of a horizontal line</td>
<td></td>
<td>$y = k$</td>
</tr>
<tr>
<td>4 slope of a line that starts in Quadrant III and ends in Quadrant I</td>
<td></td>
<td>positive</td>
</tr>
<tr>
<td>5 slope of a line that starts in Quadrant II and ends in Quadrant IV</td>
<td></td>
<td>negative</td>
</tr>
<tr>
<td>6 slope of a vertical line</td>
<td></td>
<td>undefined</td>
</tr>
<tr>
<td>7 equation of a vertical line</td>
<td></td>
<td>$x = k$</td>
</tr>
<tr>
<td>8 slopes of parallel lines</td>
<td></td>
<td>same slopes</td>
</tr>
<tr>
<td>9 slopes of perpendicular lines</td>
<td></td>
<td>opposite reciprocal slopes</td>
</tr>
<tr>
<td>10 point-slope form of equation of line</td>
<td></td>
<td>$y - y_1 = m(x - x_1)$</td>
</tr>
<tr>
<td>11 $y$-intercept form of equation of line</td>
<td></td>
<td>$y = mx + b$</td>
</tr>
<tr>
<td>12 standard form of equation of line</td>
<td></td>
<td>$Ax + By = C$ no fractions</td>
</tr>
<tr>
<td>13 distance formula</td>
<td></td>
<td>$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ or $\sqrt{(\Delta x)^2 + (\Delta y)^2}$</td>
</tr>
<tr>
<td>14 midpoint formula</td>
<td></td>
<td>$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$</td>
</tr>
</tbody>
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<td>8)</td>
</tr>
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<td></td>
<td>Find the slope of the line between the two points ((-2, 6)) and ((9, 4))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>*(Solution: (\frac{2}{11}))</td>
<td></td>
</tr>
<tr>
<td>2)</td>
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<td>9)</td>
</tr>
<tr>
<td></td>
<td>Find the slope of the line (y = 3)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>*(Solution: (m = 0))</td>
<td></td>
</tr>
<tr>
<td>3)</td>
<td>equation of a horizontal line</td>
<td>10)</td>
</tr>
<tr>
<td></td>
<td>Find the equation of the horizontal line through the point ((2, 4))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>*(Solution: (y = 4))</td>
<td></td>
</tr>
<tr>
<td>4)</td>
<td>slope of a line that starts in Quadrant III and ends in Quadrant I</td>
<td>11)</td>
</tr>
<tr>
<td></td>
<td>Find the slope of a line between the points ((-2, -5)) and ((4, 13))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>*(Solution: (m = 3))</td>
<td></td>
</tr>
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<td>5)</td>
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<td></td>
<td>Find the slope of a line between the points ((-2, 5)) and ((4, -13))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>*(Solution: (m = -3))</td>
<td></td>
</tr>
<tr>
<td>6)</td>
<td>slope of a vertical line</td>
<td>13)</td>
</tr>
<tr>
<td></td>
<td>Find the slope of the line (x = 8)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>*(Solution: (m) is undefined)</td>
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</tr>
<tr>
<td></td>
<td>Find the equation of the vertical line through the point ((2, 4))</td>
<td></td>
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<tr>
<td></td>
<td>*(Solution: (x = 2))</td>
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Translating Graphs of Lines

The following graphs are transformations of the parent function \( f(x) = x \) in the form \( f(x) = a(x \pm h) \pm k \). Set your calculator window as shown and graph each set of lines on the same screen and sketch below. Discuss the changes in the equation and what affect the change has on the graph.

1. \( f(x) = 2(x - 0) + 0 \)
   \( g(x) = 2(x + 3) + 0 \)
   \( h(x) = 2(x - 4) + 0 \)
2. \( f(x) = 2(x - 0) + 0 \)
   \( g(x) = 2(x - 0) + 3 \)
   \( h(x) = 2(x - 0) - 4 \)

3. What happens to the graph when you add a number in the function? (i.e. \( f(x + h) \))
4. What happens to the graph when you subtract a number in the function? (i.e. \( f(x - h) \))
5. What happens to the graph when you add a number to the function? (i.e. \( f(x) + k \))
6. What happens to the graph when you subtract a number from the function? (i.e. \( f(x) - k \))

7. \( f(x) = 1(x - 0) - 2 \)
   \( g(x) = \frac{1}{4}(x - 0) - 2 \)
   \( h(x) = 4(x - 0) - 2 \)
8. \( f(x) = 2(x - 0) + 3 \)
   \( g(x) = 2(-x - 0) + 3 \)

9. What happens to the graph when the function is multiplied by a number between 0 and 1? (i.e. \( k f(x) \) where \( 0 < k < 1 \))
10. What happens to the graph when the function is multiplied by a number greater than 1? (i.e. \( k f(x) \) where \( k > 1 \))
11. What happens to the graph when you take the opposite of the \( x \) in the function? (i.e. \( f(-x) \))
Translating Graphs of Lines

The following graphs are transformations of the parent function \( f(x) = x \) in the form \( f(x) = a(x \pm h) \pm k \). Set your calculator window as shown and graph each set of lines on the same screen and sketch below. Discuss the changes in the equation and what affect the change has on the graph.

1. \( f(x) = 2(x - 0) + 0 \)
   \( g(x) = 2(x + 3) + 0 \)
   \( h(x) = 2(x - 4) + 0 \)

2. \( f(x) = 2(x - 0) + 0 \)
   \( g(x) = 2(x - 0) + 3 \)
   \( h(x) = 2(x - 0) - 4 \)

3. What happens to the graph when you add a number in the function? (i.e. \( f(x + h) \))
   The graph moves to the left.

4. What happens to the graph when you subtract a number in the function? (i.e. \( f(x - h) \))
   The graph moves to the right.

5. What happens to the graph when you add a number to the function? (i.e. \( f(x) + k \))
   The graph moves up.

6. What happens to the graph when you subtract a number from the function? (i.e. \( f(x) - k \))
   The graph moves down.

7. \( f(x) = \frac{1}{4}(x - 0) \)
   \( g(x) = \frac{1}{4}(x - 0) \)
   \( h(x) = 4(x - 0) \)

8. \( f(x) = 2(x - 0) + 3 \)
   \( g(x) = 2(-x - 0) + 3 \)

9. What happens to the graph when the function is multiplied by a number between 0 and 1? (i.e. \( k f(x) \) where \( 0 < k < 1 \))
   It becomes less steep.

10. What happens to the graph when the function is multiplied by a number greater than 1? (i.e. \( k f(x) \) where \( k > 1 \))
    It becomes steeper.

11. What happens to the graph when you take the opposite of the x in the function? (i.e. \( f(-x) \))
    It rotates the graph on the y-axis.
Graphing Absolute Value Functions

Graph the following piecewise function by hand:

(1) \( f(x) = \begin{cases} 
  x & \text{if } x \geq 0 \\
  -x & \text{if } x < 0 
\end{cases} \)

(2) On your graphing calculator graph the function \( f(x) = |x| \) with this WINDOW and answer the following questions. (Note: Absolute value is under MATH > NUM > 1: abs(, so in your calculator you will type \( y_1 = \text{abs}(x) \))

a. Compare the graph to the graph in #1 above. What is the relationship between the two?
b. What is the shape of the graph?
c. What is the slope of the two lines that create the graph?
d. What is the vertex of the graph?
e. What is the domain and range?
f. What is the axis of symmetry?

Translating Graphs of Absolute Value Functions

The following graphs are transformations of the parent function \( f(x) = |x| \) in the form \( f(x) = a|x - h| + k \). Graph each on your calculator and sketch below and observe the type of transformation.

(3) \( f(x) = |x| - 4 \)

(4) \( f(x) = |x| + 2 \)

(5) What happens to the graph when you subtract a number from the function? (i.e. \( f(x) - k \))

(6) What happens to the graph when you add a number to the function? (i.e. \( f(x) + k \))
Unit 1, Activity 7, Translating Absolute Value Functions Discovery Worksheet

(7) \(f(x) = |x - 4|\)  
\(f(x) = |x + 2|\)

(8) 

(9) What happens to the graph when you subtract a number in the function? (i.e. \(f(x - h)\))

(10) What happens to the graph when you add a number in the function? (i.e. \(f(x + h)\))

(11) \(f(x) = -|x|\)

(12) What happens to the graph when you take the opposite of the function? (i.e. \(-f(x)\))

(13) \(f(x) = 2|x|\)

(14) \(f(x) = \frac{5}{2}|x|\)

(15) \(f(x) = \frac{1}{2}|x|\)

(16) \(f(x) = \frac{2}{5}|x|\)

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Louisiana Comprehensive Curriculum, Revised 2008
17) What happens to the graph when the function is multiplied by a number greater than 1?

(18) What happens to the graph when the function is multiplied by a number between 0 and 1?

(19) These graphs are in the form \( af(x) \). What does the “\( a \)” represent in these graphs?

**Synthesis** Write an equation for each described transformation.

(20) a V-shape shifted down 4 units: \( f(x) = \) ________________

(21) a V-shape shifted left 6 units: \( f(x) = \) ________________

(22) a V-shape shifted right 2 units and up 1 unit: \( f(x) = \) ________________

(23) a V-shape flipped upside down and shifted left 5 units: \( f(x) = \) ________________

**Analysis** Describe the transformation that has taken place for the parent function \( f(x) = |x| \).

(24) \( f(x) = |x| - 5 \) ________________

(25) \( f(x) = 5|x + 7| \) ________________

(26) \( f(x) = -\frac{1}{4}|x| \) ________________

(27) \( f(x) = |x - 4| + 3 \) ________________

(28) Graph the function \( f(x) = 2|x - 1| - 3 \) without a calculator and answer the following questions:

a. What is the shape of the graph?

b. What is the vertex of the graph and how do you know?

c. Does it open up or down and how do you know?

d. What are the slopes of the two lines that create the graph?

e. What is the domain and range?

f. What is the axis of symmetry?
Graphing Absolute Value Functions

Graph the following piecewise function by hand:

\[ f(x) = \begin{cases} 
  x & \text{if } x \geq 0 \\
  -x & \text{if } x < 0
\end{cases} \]

On your graphing calculator graph the function \( f(x) = |x| \) with this WINDOW and answer the following questions. (Note: Absolute value is under MATH > NUM > 1: abs( so in your calculator you will type \( y_1 = \text{abs}(x) \))

a. Compare the graph to the graph in #1 above. What is the relationship between the two? the graphs are the same
b. What is the shape of the graph? Two rays with a common endpoint that form a V.
c. What is the slope of the two lines that create the graph? \( m = \pm 1 \)
d. What is the vertex of the graph? \((0, 0)\)
e. What is the domain and range? Domain: all reals, Range: \( y > 0 \)
f. What is the axis of symmetry? \( x = 0 \)

Translating Graphs of Absolute Value Functions

The following graphs are transformations of the parent function \( f(x) = |x| \) in the form \( f(x) = a|x - h| + k \). Graph each on your calculator and sketch below and observe the type of transformation.

\[
(3) \quad f(x) = |x| - 4 \\
(4) \quad f(x) = |x| + 2
\]

(5) What happens to the graph when you subtract a number from the function? (i.e. \( f(x) - k \))
The graph shifts down.

(6) What happens to the graph when you add a number to the function? (i.e. \( f(x) + k \))
The graph shifts up.
(7) \( f(x) = |x - 4| \)

(8) \( f(x) = |x + 2| \)

(9) What happens to the graph when you subtract a number \( h \) in the function? (i.e. \( f(x - h) \))
The graph shifts to the right.

(10) What happens to the graph when you add a number \( h \) in the function? (i.e. \( f(x + h) \))
The graph shifts to the left.

(11) \( f(x) = -|x| \)

(12) What happens to the graph when you take the opposite of the function? (i.e. \( -f(x) \))
The graph rotates on the \( x \)-axis.

(13) \( f(x) = 2|x| \)

(14) \( f(x) = \frac{5}{2}|x| \)

(15) \( f(x) = \frac{1}{2}|x| \)

(16) \( f(x) = \frac{2}{5}|x| \)
Unit 1, Activity 7, Translating Absolute Value Functions Discovery Worksheet with Answers

(17) What happens to the graph when the function is multiplied by a number greater than 1?  
*It gets steeper.*

(18) What happens to the graph when the function is multiplied by a number between 0 and 1?  
*It gets less steep.*

(19) These graphs are in the form $af(x)$. What does the “$a$” represent in these graphs?  
The slopes of the two rays are $\pm a$.

**Synthesis** Write an equation for each described transformation.

(20) a V-shape shifted down 4 units:  
$$f(x) = |x| - 4$$

(21) a V-shape shifted left 6 units:  
$$f(x) = |x + 6|$$

(22) a V-shape shifted right 2 units and up 1 unit:  
$$f(x) = |x - 2| + 1$$

(23) a V-shape flipped upside down and shifted left 5 units:  
$$f(x) = -|x + 5|$$

**Analysis** Describe the transformation that has taken place for the parent function $f(x) = |x|$.

(24) $f(x) = |x| - 5$  
a V-shaped graph shifted down 5 units

(25) $f(x) = 5|x + 7|$  
a steeper (slopes of $\pm 5$) V-shaped graph shifted left 7 units

(26) $f(x) = -\frac{1}{4}|x|$  
an upside down V-shaped graph not very steep – slopes of $\pm \frac{1}{4}$

(27) $f(x) = |x - 4| + 3$  
a V-shaped graph shifted right 4 and up 3

(28) Graph the function $f(x) = 2|x - 1| - 3$ without a calculator and answer the following questions:
  
a. What is the shape of the graph? V-shaped

  b. What is the vertex of the graph and how do you know?  
     *(1, −3) because it shifted right 1 and down 3.*

  c. Does it open up or down and how do you know?  
     *up because the leading coefficient is positive.*

  d. What are the slopes of the two lines that create the graph? $m = \pm 2$

  e. What is the domain and range? Domain: all reals, Range: $y > -3$

  f. What is the axis of symmetry? $x = 1$
**Unit 1, Activity 9, Absolute Value Inequalities Discovery Worksheet**

You previously learned how to solve one variable absolute value inequality \(|x - h| > d\) using the concept that \(x\) is greater than a distance of \(d\) from the center \(h\), and to also write the answer as an “and” or “or” statement. In this activity you will discover how to use the graph of a two variable absolute value function \(y = |x - h| + k\) to help you solve a one variable absolute value inequality.

1. Solve the inequality \(|x - 4| > 5\). Write the solution in terms of “distance” and in interval notation.

\[\text{solution in interval notation: } \]  

\[\text{Explanation: } \]

2. Isolate zero in the equation \(|x - 4| > 5\).

3. Graph the function \(f(x) = |x - 4| - 5\)

4. Write the function as a piecewise function.

5. Find the zeroes of each piece of the piecewise function.

6. Use the graph of \(f(x)\) to determine the interval(s) where \(f(x) > 0\) and explain how you got the answer looking at the graph. Does your answer match the answer to #1?

\[\text{solution in interval notation: } \]  

\[\text{Explanation: } \]

7. Write the solution in #6 in set notation. Using the piecewise function for \(f(x)\) in #4, explain why the solution to \(|x - 4| > 5\) is an “or” statement instead of an “and” statement.

\[\text{solution in set notation: } \]  

\[\text{Explanation: } \]
Unit 1, Activity 9, Absolute Value Inequalities Discovery Worksheet

Practice

(8) Graph \( f(x) = |x + 2| - 6 \) and write \( f(x) \) as a piecewise function and find the zeroes.

\[
f(x) = \begin{cases} 
    \text{zeroes: } x = \quad \text{and } x = \\
\end{cases}
\]

Solve \(|x + 2| - 6 \leq 0\) using the graph above (interval notation)

(9) Graph \( f(x) = 3|x - 4| - 6 \) and write \( f(x) \) as a piecewise function.

\[
f(x) = \begin{cases} 
    \text{zeroes: } x = \quad \text{and } x = \\
\end{cases}
\]

Solve \(3|x - 4| - 6 > 0\) using the graph above (interval notation)

(10) Graph \( f(x) = -2|x - 3| + 8 \) and write \( f(x) \) as a piecewise function.

\[
f(x) = \begin{cases} 
    \text{zeroes: } x = \quad \text{and } x = \\
\end{cases}
\]

Solve \(-2|x - 3| + 8 \geq 0\) using the graph above (interval notation)
Unit 1, Activity 9, Absolute Value Inequalities Discovery Worksheet with Answers

Name________________________________________ Date________________

**Absolute Value Inequalities** You previously learned how to solve the one variable absolute value inequality \(|x - h| > d\) using the concept that \(x\) is greater than a distance of \(d\) from the center \(h\), and to also write the answer as an “and” or “or” statement. In this activity you will discover how to use the graph of a two variable absolute value function \(y = |x - h| + k\) to help you solve a one variable absolute value inequality.

1. Solve the inequality \(|x - 4| > 5\). Write the solution in terms of “distance” and in interval notation.

   \(x\) is a distance greater than 5 from 4 so the interval solution is \((-\infty, -1) \cup (9, \infty)\).

2. Isolate zero in the equation \(|x - 4| > 5\). \(|x - 4| - 5 > 0\)

3. Graph the function \(f(x) = |x - 4| - 5\)

4. Write the function as a piecewise function.

   \[f(x) = \begin{cases} 
   x - 9 & \text{if } x \geq 4 \\
   -x - 1 & \text{if } x < 4
   \end{cases}\]

5. Find the zeroes of each piece of the piecewise function.

   zeroes at \(x = -1\) and 9

6. Use the graph of \(f(x)\) to determine the interval/s where \(f(x) > 0\) and explain how you got the answer looking at the graph. Does your answer match the answer to #1?

   solution in interval notation: \((-\infty, -1) \cup (9, \infty)\). Explanation: By looking at the graph and the zeroes, you can find the values of \(x\) for which the \(y\) values are > 0

7. Write the solution in #6 in set notation. Using the piecewise function for \(f(x)\) in #4, explain why the solution to \(|x - 4| > 5\) is an “or” statement instead of an “and” statement.

   solution in set notation: \(\{x : x < -1 \text{ or } x > 9\}\). Explanation: Since the intervals are in different equations of the piecewise function and do not intersect, you must use union which is an “or” statement.
(8) Graph \( f(x) = |x + 2| - 6 \) and write \( f(x) \) as a piecewise function and find the zeroes.

\[
f(x) = \begin{cases} 
  x - 4 & \text{if } x \geq -2 \\
  -x - 8 & \text{if } x < -2
\end{cases}
\]

zeroes: \( x = 4 \) and \(-8\),

Solve \( |x + 2| - 6 \leq 0 \) using the graph above (interval notation) \([-8, 4]\)

(9) Graph \( f(x) = 3|x - 4| - 6 \) and write \( f(x) \) as a piecewise function.

\[
f(x) = \begin{cases} 
  3x - 18 & \text{if } x \geq 4 \\
  -3x + 6 & \text{if } x < 4
\end{cases}
\]

zeroes: \( x = 6 \) and \(2\)

Solve \( 3|x - 4| - 6 > 0 \) using the graph above (interval notation) \((\infty, 4) \cup (6, \infty)\)

(10) Graph \( f(x) = -2|x - 3| + 8 \) and write \( f(x) \) as a piecewise function.

\[
f(x) = \begin{cases} 
  -2x + 14 & \text{if } x \geq 3 \\
  2x + 2 & \text{if } x < 3
\end{cases}
\]

zeroes: \( x = -1 \) and \(7\)

Solve \( -2|x - 3| + 8 \geq 0 \) using the graph above (interval notation) \([-1, 7]\)
Graphing the Greatest Integer Function

(1) On the graphing calculator, graph \( y = \text{int}(x) \). (Note: On the TI calculator, the greatest integer function is under MATH, NUM, 5: int( . In your calculator you will type \( y_1 = \text{int}(x) \).) If it looks like the first graph below, the calculator is in connected mode. Change the mode to dot mode under MODE, DOT.

![Graphs](image)

(2) Write a piecewise function for the graph above on the domain \(-3 < x < 3\) and state the range.

\[
f(x) =
\begin{cases}
  & \\
  & \\
  & 
\end{cases}
\]

Range: _______________________

The above piecewise function is defined symbolically as \( f(x) = \lfloor x \rfloor \) and verbally as “the greatest integer less than or equal to \( x \)” or, in other words, a “round down” function. It is a step function, and the graph is said to have “jump discontinuities” at the integers.

Evaluating Greatest Integer Expressions

Evaluate the following:

(3) \( \lfloor 7.1 \rfloor = \) ________

(4) \( \lfloor 1.8 \rfloor = \) ________

(5) \( \lfloor \pi \rfloor = \) ________

(6) \( \lfloor -6.8 \rfloor = \) ________

(7) \( \lfloor -2.1 \rfloor = \) ________

(8) \( \lfloor 0 \rfloor = \) ________

Solving Greatest Integer Equations

Solve the following equations for \( x \) and write the answers in set notation:

(9) \( \lfloor \frac{2x}{7} \rfloor = 1 \)

(10) \( \lfloor 3x \rfloor = 12 \)
Translating Graphs of Greatest Integer Functions

Using what you learned about the translations of \( y = a|x - h| + k \), graph the following by hand and check on your calculator:

(11) \( f(x) = \lfloor x \rfloor + 2 \) \hspace{1cm} \( g(x) = \lfloor x + 2 \rfloor \)

Explain the shift in each graph and how they differ.

(12) \( f(x) = 2\lfloor x \rfloor \) \hspace{1cm} \( g(x) = \lfloor 2x \rfloor \)

Explain the dilation in each graph and how they differ.

(13) \( f(x) = -\lfloor x \rfloor \) \hspace{1cm} \( g(x) = \lfloor -x \rfloor \)

Explain the reflection in these graphs and how they differ.
Real World Application of Step Functions

Prior to September, 2000, taxi fares from Washington DC to Maryland were described as follows: $2.00 up to and including $\frac{1}{2}$ mile, $0.70$ for each additional $\frac{1}{2}$ mile increment.

(14) Describe the independent and dependent variables and explain your choices. ________________

(15) Graph the fares for the first 2 miles: (Make sure to label the axes.)

(16) Write the piecewise function for 0 to 2 miles.

\[
f(x) =
\begin{cases}
2 & \text{if } 0 \leq x < 0.5, \\
0.7(x - 0.5) + 2 & \text{if } 0.5 \leq x < 1, \\
2 & \text{if } 1 \leq x < 1.5, \\
0.7(x - 1.5) + 2 & \text{if } 1.5 \leq x < 2, \\
2 & \text{if } 2 \leq x.
\end{cases}
\]

(17) Discuss why this is a step function and it is different from the greatest integer parent function \( f(x) = \lfloor x \rfloor \).
Unit 1, Activity 10, Greatest Integer Discovery Worksheet with Answers

Name__________________________________________ Date____________________

Graphing the Greatest Integer Function

(1) On the graphing calculator, graph \( y = \text{int}(x) \). (Note: On the TI calculator, the greatest integer function is under MATH, NUM, 5: int(. In your calculator you will type \( y_1 = \text{int}(x) \).) If it looks like the first graph below, the calculator is in connected mode. Change the mode to dot mode under MODE, DOT.

(2) Write a piecewise function for the graph above on the domain \(-3 < x < 3\) and state the range.

\[
\begin{align*}
    f(x) = & \begin{cases}
        3 & \text{if } x = 3 \\
        2 & \text{if } 2 \leq x < 3 \\
        1 & \text{if } 1 \leq x < 2 \\
        0 & \text{if } 0 \leq x < 1 \\
        -1 & \text{if } -1 \leq x < 0 \\
        -2 & \text{if } -2 \leq x < -1 \\
        -3 & \text{if } x = -3
    \end{cases}
\end{align*}
\]

The above piecewise function is defined symbolically as \( f(x) = \left\lfloor x \right\rfloor \) and verbally as “the greatest integer less than or equal to \( x \)” or, in other words, a “round down” function. It is a step function, and the graph is said to have “jump discontinuities” at the integers.

Evaluating Greatest Integer Expressions

Evaluate the following:

(3) \( \left\lfloor 7.1 \right\rfloor = 7 \) \hspace{1cm} (4) \( \left\lfloor 1.8 \right\rfloor = 1 \) \hspace{1cm} (5) \( \left\lfloor \pi \right\rfloor = 3 \)
(6) \( \left\lfloor -6.8 \right\rfloor = -7 \) \hspace{1cm} (7) \( \left\lfloor -2.1 \right\rfloor = -3 \) \hspace{1cm} (8) \( \left\lfloor 0 \right\rfloor = 0 \)

Solving Greatest Integer Equations

Solve the following equations for \( x \) and write the answers in set notation:

(9) \( \left\lfloor \frac{2x}{7} \right\rfloor = 1 \) \hspace{1cm} (10) \( \left\lfloor 3x \right\rfloor = 12 \)

Solution: \( \frac{7}{2} \leq x < 7 \) \hspace{1cm} Solution: \( 4 \leq x < \frac{13}{3} \)
Translating Graphs of Greatest Integer Functions

Using what you learned about the translations of \( y = a|x - h| + k \), graph the following by hand and check on your calculator:

\[
(11) \quad f(x) = \left\lfloor x \right\rfloor + 2 \quad \quad \quad g(x) = \left\lfloor x + 2 \right\rfloor.
\]

Explain the shift in each graph and how they differ. **In** \( f(x) \) the \( y \) values were shifted up 2 but in \( g(x) \) the \( x \) values were shifted to the left 2; however, the results were the same.

\[
(12) \quad f(x) = 2\left\lfloor x \right\rfloor \quad \quad \quad g(x) = \left\lfloor 2x \right\rfloor
\]

Explain the dilation in each graph and how they differ. **The** \( y \) values of \( f(x) \) were multiplied by 2 while the \( x \) values in \( g(x) \) were divided by 2.

\[
(13) \quad f(x) = -\left\lfloor x \right\rfloor \quad \quad \quad g(x) = \left\lfloor -x \right\rfloor
\]

Explain the reflection in these graphs and how they differ. **In** \( f(x) \) the \( y \) values were rotated around the \( x \)-axis and in the \( g(x) \) the \( x \) values were rotated around the \( y \)-axis; however the results were the same.
Real World Application of Step Functions

Prior to September, 2000, taxi fares from Washington DC to Maryland were described as follows: $2.00 up to and including ½ mile, $0.70 for each additional ½ mile increment.

(14) Describe the independent and dependent variables and explain your choices. The number of miles is the independent variable and the fare is the dependent variable because the fare depends on how far you travel.

(15) Graph the fares for the first 2 miles: (Make sure to label the axes.)

(16) Write the piecewise function for 0 to 2 miles.

\[ f(x) = \begin{cases} 
2.00 & \text{if } 0 < x \leq 0.5 \text{ mile} \\
2.70 & \text{if } 0.5 < x \leq 1 \text{ mile} \\
3.40 & \text{if } 1 < x \leq 1.5 \text{ miles} \\
4.10 & \text{if } 1.5 < x \leq 2 \text{ miles} 
\end{cases} \]

(17) Discuss why this is a step function and it is different from the greatest integer parent function. This is a step function because it is made with horizontal line segments. It is different than the greatest integer function because it does not start at 0, jump discontinuities occur at every increment of ½ instead of 1, and the increments of y are .7 instead of 1. It also rounds up instead of rounding down meaning the closed dot is on the right side of the horizontal step instead of on the left.
Unit 1, Activity 10, Step Function Data Research Project

Name______________________________ Date____________________

Objective: To find data on the Internet or in the newspaper that is conducive to creating a step function graph.

Materials: ½ piece of poster board, colored pencils, markers or crayons, data

Directions:
- Find data on the Internet or in newspapers or real-life situations that are indicative of step functions. Make sure to write down your source. Write the data in a clearly labeled table on the poster board.
- Using the data, draw a step function graph with the axis clearly labeled.
- Determine the piecewise equation for the step function including the domain in the equation. Specify the overall domain and range of the function.
- Write a real world problem in which the data can be used to interpolate and extrapolate to solve another problem. Do the interpolation and extrapolation and find the correct answer. Discuss if this answer is realistic.

The following information must be on the front of the poster board: (everything must be in color – be creative)

1. Creative title (keep it clean) with poster neat, complete, readable, and decorated relative to the topic.
2. Data neatly presented in a clearly labeled table with source included.
3. Graph of the step function showing the x and y axes and units of measure.
4. Piecewise function with domain and range
5. Real world word problem interpolating and extrapolating and solved correctly.
6. Your name, date and hour
# Unit 1, Activity 10, Step Function Data Research Project Grading Rubric

## Rubric: Step Function Data Research Project

<table>
<thead>
<tr>
<th>CATEGORY &amp; Sub Score</th>
<th>10–8</th>
<th>7–5</th>
<th>4–2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td>Professional looking and accurate representation of the data in tables. Tables are labeled and titled. Source of information is included.</td>
<td>Accurate representation of the data in tables. Tables are labeled and titled. Source of information is included.</td>
<td>Accurate representation of the data in written form, but no tables are presented. Source of information is missing.</td>
<td>Data is not shown or is not step function data OR is inaccurate.</td>
</tr>
<tr>
<td><strong>Step Function Graph</strong></td>
<td>Clear, accurate step function graph is included and makes the research easier to understand. Graph is labeled neatly and accurately.</td>
<td>Graph is included and is labeled neatly and accurately.</td>
<td>Graph is included and is not labeled.</td>
<td>Needed graph is missing OR is missing important labels.</td>
</tr>
<tr>
<td><strong>Piecewise Equation, Domain and Range</strong></td>
<td>Piecewise equation with domain and range is accurate and symbolically correct.</td>
<td>Piecewise equation with domain and range is accurate but symbolically incorrect.</td>
<td>Piecewise equation is correct but domain and range are missing.</td>
<td>No piecewise equation is shown OR results are inaccurate or mislabeled.</td>
</tr>
<tr>
<td><strong>Real World Problem</strong></td>
<td>Real life problem is included and typed and uses the function to interpolate and extrapolate and has correct answers.</td>
<td>Real life problem is included typed or handwritten and uses the function to interpolate or extrapolate and has correct answers.</td>
<td>Real life problem is written and uses the function to interpolate or extrapolate but answers are incorrect.</td>
<td>Real life problem is handwritten but no interpolation or extrapolation.</td>
</tr>
<tr>
<td><strong>Poster</strong></td>
<td>Poster is neat, complete and creative and uses headings and subheadings to visually organize the material. Poster is decorated relative to the topic.</td>
<td>Poster is neat and complete and material is visually organized but not decorated relative to the topic.</td>
<td>Poster is neat but incomplete.</td>
<td>Poster is handwritten and looks sloppy and is incomplete</td>
</tr>
</tbody>
</table>
Composite Functions in a Double Function Machine

(1) \( f(x) = 3x + 7 \) and \( g(x) = -4x - 1 \). Find \( f(g(5)) \) and \( g(f(5)) \) with the function machine.

\[
\begin{array}{c|c|c}
 x & g(x) & f(x) \\
\hline
 5 & -4x - 1 & 3x + 7 \\
\end{array}
\]

\( f(g(5)) = \ldots \) \( g(f(5)) = \ldots \)

(2) Using the following function machine to find a rule for \( f(g(x)) \) and \( g(f(x)) \).

\[
\begin{array}{c|c|c}
 x & g(x) & f(x) \\
\hline
 x & -4x - 1 & 3x + 7 \\
\end{array}
\]

\( f(g(x)) = \ldots \) \( g(f(x)) = \ldots \)

Finding Equations of Composite Functions and Graphing Them on the Calculator

- In order to graph the composition \( f(g(x)) \) on a graphing calculator, enter \( g(x) \) into \( y_1 = -4x - 1 \) and turn it off so it will not graph. (Note: To turn an equation off, use your left arrow to move the cursor over the = sign and press ENTER.)
- Next, enter \( f(x) \) into \( y_2 \) as follows \( y_2 = 3(y_1) + 7 \) and graph. (Note: \( y_1 \) is under \( \Theta, Y-VARS, 1: \) Function, 1: \( Y_1 \).)
- Graph the answer to \( f(g(x)) \) from the function machine in #2 above in \( y_3 \), to see if they are the same graph.

(3) Practice with the polynomial functions: \( f(x) = 2x + 1 \) and \( g(x) = 4x^2 + 3 \). Find \( f(g(x)) \) and \( g(f(x)) \) and check on the calculator.

\[
\begin{array}{c|c|c}
 x & g(x) & f(x) \\
\hline
 x & 4x^2 + 3 & 2x + 1 \\
\end{array}
\]

\( f(g(x)) = \ldots \) \( g(f(x)) = \ldots \)
**Synthesis of Composite Functions** Use \( f(x) = 3x^2 + 2 \) to evaluate the functions in #4 and #5 and to create new functions in #6 – 8:

(4) \( f(3) = \) ____________________________

(5) \( f(a) = \) ____________________________

(6) \( f(a + b) = \) ____________________________

(7) \( f(x) + h = \) ____________________________

(8) \( f(x + h) = \) ____________________________

(9) One of the most difficult compositions that is also very necessary for higher mathematics is finding \( \frac{f(x + h) - f(x)}{h} \) which is called a difference quotient. Find the difference quotient for \( f(x) = 3x^2 + 2 \)

(10) Find the difference quotient \( \frac{g(x + h) - g(x)}{h} \) for \( g(x) = -2x - 5 \).

**Composite Functions in a Table** Use the table to calculate the following compositions:

(11) \( f(g(2)) = \) ____________________________

(12) \( g(f(2)) = \) ____________________________

(13) \( f(g(3)) = \) ____________________________

(14) \( g(f(3)) = \) ____________________________

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>
Unit 1, Activity 11, Composite Function Discovery Worksheet

Decomposition of Composite Functions

Most functions are compositions of basic functions. Work backwards to determine the basic functions that created the composition.

<table>
<thead>
<tr>
<th></th>
<th>$f(g(x))$</th>
<th>$f(x)$</th>
<th>$g(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(15)</td>
<td>$(x + 4)^2 + 5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(16)</td>
<td>$\sqrt{x - 4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(17)</td>
<td>$(4x - 1)^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(18)</td>
<td>$</td>
<td>x + 2</td>
<td>$</td>
</tr>
<tr>
<td>(19)</td>
<td>$\lceil x - 2 \rceil + 4$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Domain & Range of Composite Functions

Find the domains and composition $f(g(x))$ to fill in the table below to discover the rule for the domain of a composite function:

<table>
<thead>
<tr>
<th></th>
<th>$f(x)$</th>
<th>$g(x)$</th>
<th>Domain of $f(x)$</th>
<th>Domain of $g(x)$</th>
<th>$f(g(x))$</th>
<th>Domain of $f(g(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(20)</td>
<td>$\sqrt{x - 3}$</td>
<td>$x + 1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(21)</td>
<td>$x + 1$</td>
<td>$\sqrt{x - 3}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(22)</td>
<td>$\frac{1}{x}$</td>
<td>$2x + 4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(23)</td>
<td>$2x + 4$</td>
<td>$\frac{1}{x}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(24)</td>
<td>$\sqrt{x}$</td>
<td>$x^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(25)</td>
<td>$x^2$</td>
<td>$\sqrt{x}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(26) Develop a rule for determining the domain of a composition: __________________________

______________________________

______________________________
Composite Functions in a Double Function Machine

(1) \( f(x) = 3x + 7 \) and \( g(x) = -4x - 1 \). Find \( f(g(5)) \) and \( g(f(5)) \) with the function machine.

\[
\begin{array}{c|c|c|c}
  x = 5 & g(x) = -4x - 1 & 21 & f(x) = 3x + 7 & -56 \\
  x = 5 & f(x) = 3x + 7 & 22 & g(x) = -4x - 1 & -89 \\
\end{array}
\]

\[ f(g(5)) = -56 \quad g(f(5)) = -89 \]

(2) Using the following function machine to find a rule for \( f(g(x)) \) and \( g(f(x)) \).

\[
\begin{array}{c|c|c|c|c|c}
  x = x & g(x) = -4x - 1 & -4x - 1 & f(x) = 3x + 7 & 3(-4x - 1) + 7 \\
  x = x & f(x) = 3x + 7 & 3x + 7 & g(x) = -4x - 1 & -4(3x + 7) - 1 \\
\end{array}
\]

\[ f(g(x)) = -12x + 4 \quad g(f(x)) = -12x - 29 \]

Finding Equations of Composite Functions and Graphing Them on the Calculator

- In order to graph the composition \( f(g(x)) \) on a graphing calculator, enter \( g(x) \) into \( y_1 = -4x - 1 \) and turn it off so it will not graph. (Note: To turn an equation off, use your left arrow to move the cursor over the = sign and press ENTER.)
- Next, enter \( f(x) \) into \( y_2 \) as follows \( y_2 = 3(y_1) + 7 \) and graph. (Note: \( y_1 \) is under \( \Theta \), \( Y-VARS \), 1: Function, 1: \( Y_1 \).)
- Graph the answer to \( f(g(x)) \) from the function machine in #2 above in \( y_3 \), to see if they are the same graph.

(3) Practice with the polynomial functions: \( f(x) = 2x + 1 \) and \( g(x) = 4x^2 + 3 \). Find \( f(g(x)) \) and \( g(f(x)) \) and check on the calculator.

\[
\begin{array}{c|c|c|c|c|c}
  x = x & g(x) = 4x^2 + 3 & 4x^2 + 3 & f(x) = 2x + 1 & 2(4x^2 + 3) + 1 \\
  x = x & f(x) = 2x + 1 & 2x + 1 & g(x) = 4x^2 + 3 & 4(2x + 1)^2 + 3 \\
\end{array}
\]

\[ f(g(x)) = 8x^2 + 7 \quad g(f(x)) = 16x^2 + 16x + 7 \]
Synthesis of Composite Functions

Use \( f(x) = 3x^2 + 2 \) to evaluate the function in #4 and #5 and to create a composite function in #6 – 8:

(4) \( f(3) = \underline{29} \)

(5) \( f(a) = \underline{3a^2 + 2} \)

(6) \( f(a + b) = \underline{3a^2 + 6ab + 3b^2 + 2} \)

(7) \( f(x) + h = \underline{3x^2 + 2 + h} \)

(8) \( f(x+h) = \underline{3x^2 + 6xh + 3h^2 + 2} \)

(9) One of the most difficult compositions, that is also very necessary for higher mathematics, is finding \( \frac{f(x + h) - f(x)}{h} \) which is called a difference quotient. Find the difference quotient for \( f(x) = 3x^2 + 2 \)

Solution: \( 6x + 3h \)

(10) Find the difference quotient \( \frac{g(x + h) - g(x)}{h} \) for \( g(x) = -2x - 5 \).

Solution: \( -2 \)

Composite Functions in a Table

Use the table to calculate the following compositions:

(11) \( f(g(2)) = \underline{3} \)

(12) \( g(f(2)) = \underline{12} \)

(13) \( f(g(3)) = \underline{9} \)

(14) \( g(f(3)) = \text{cannot be determined} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
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<td>5</td>
<td>3</td>
<td>7</td>
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</table>
Decomposition of Composite Functions

Most functions are compositions of basic functions. Work backwards to determine the basic functions that created the composition.

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<th>f(x)</th>
<th>g(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(15)</td>
<td>(x + 4)^2 + 5</td>
<td>x^2 + 5</td>
<td>x + 4</td>
</tr>
<tr>
<td>(16)</td>
<td>\sqrt{x - 4}</td>
<td>\sqrt{x}</td>
<td>x - 4</td>
</tr>
<tr>
<td>(17)</td>
<td>(4x - 1)^2</td>
<td>x^2</td>
<td>4x - 1</td>
</tr>
<tr>
<td>(18)</td>
<td></td>
<td>x</td>
<td>+ 2</td>
</tr>
<tr>
<td>(19)</td>
<td>[x - 2] + 4</td>
<td>[x] + 4</td>
<td>x - 2</td>
</tr>
</tbody>
</table>

Domain & Range of Composite Functions

Find the domains and composition \( f(g(x)) \) to fill in the table below to discover the rule for the domain of a composite function:

<table>
<thead>
<tr>
<th></th>
<th>f(x)</th>
<th>g(x)</th>
<th>Domain of f(x)</th>
<th>Domain of g(x)</th>
<th>f(g(x))</th>
<th>Domain of f(g(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(20)</td>
<td>\sqrt{x - 3}</td>
<td>x + 1</td>
<td>x ≥ 3</td>
<td>all reals</td>
<td>\sqrt{x - 2}</td>
<td>x ≥ 2</td>
</tr>
<tr>
<td>(21)</td>
<td>x + 1</td>
<td>\sqrt{x - 3}</td>
<td>all reals</td>
<td>x ≥ 3</td>
<td>\sqrt{x - 3 + 1}</td>
<td>x ≥ 3</td>
</tr>
<tr>
<td>(22)</td>
<td>\frac{1}{x}</td>
<td>2x + 4</td>
<td>x ≠ 0</td>
<td>all reals</td>
<td>\frac{1}{2x + 4}</td>
<td>x ≠ -2</td>
</tr>
<tr>
<td>(23)</td>
<td>2x + 4</td>
<td>\frac{1}{x}</td>
<td>all reals</td>
<td>x ≠ 0</td>
<td>\frac{2}{x + 4}</td>
<td>x ≠ 0</td>
</tr>
<tr>
<td>(24)</td>
<td>\sqrt{x}</td>
<td>x^2</td>
<td>x ≥ 0</td>
<td>all reals</td>
<td>\sqrt{x^2}</td>
<td>all reals</td>
</tr>
<tr>
<td>(25)</td>
<td>x^2</td>
<td>\sqrt{x}</td>
<td>all reals</td>
<td>x ≥ 0</td>
<td>(\sqrt{x})^2</td>
<td>x ≥ 0</td>
</tr>
</tbody>
</table>

(26) Develop a rule for determining the domain of a composition: To determine the domain of the composition \( f(g(x)) \), find the domain of \( g(x) \) and further restrict it for the composition \( f(g(x)) \). Note that the domain restrictions on \( f(x) \) have no consequences on the composition.
Recognizing Inverse Functions: An inverse relation is defined as any relation that swaps the independent and dependent variables. Determine whether the inverse of every function is a function by swapping the variables in each relation below to determine if the new relation is a function, and why or why not.

(1) the set of ordered pairs \{(x, y) : (1, 2), (3, 5), (3, 6), (7, 5), (8, 2)\}
   Swap the \(x\) and \(y\) and write a new set of ordered pairs: __________________________
   Is the new relation a function of \(x\)? ______ Explain why or why not. ________________

(2) the set of ordered pairs \{(s, d) : (1, 1), (2, 4), (3, 9), (–1, 1), (–2, 4), (–3, 9)\}
   Swap the \(s\) and \(d\) and write a new set of ordered pairs: __________________________
   Is the new relation a function of \(s\)? ______ Explain why or why not. ________________

(3) the relationship “\(x\) is a student of \(y\)”
   Write the words for the inverse relationship. __________________________
   Is the new relation a function of \(x\)? ______ Explain why or why not. ________________

(4) the relationship “\(x\) is the biological daughter of mother \(y\)”
   Write the words for the inverse relationship. __________________________
   Is the new relation a function of \(x\)? ______ Explain why or why not. ________________

(5) the equation \(2x + 3y = 6\)
   Swap the \(x\) and \(y\) and write a new equation: __________________________
   Is the new relation a function of \(x\)? ______ Explain why or why not. ________________

(6) the equation \(x + y^2 = 9\)
   Swap the \(x\) and \(y\) and write a new equation: __________________________
   Is the new relation a function of \(x\)? ______ Explain why or why not. ________________
Unit 1, Activity 12, Inverse Function Discovery Worksheet

(7) **the equation** \( y = x^2 + 4 \)
    Swap the \( x \) and \( y \) and write a new equation: ________________________________

    Is the new relation a function of \( x \)? _______ Explain why or why not. _________________

(8) **the graph of the circle**

    If you swapped the \( x \) and \( y \) what would the new graph look like? _________________

    Is the new graph a function of \( x \)? _______ Explain why or why not. __________________

Answer the following questions:

(9) How can you look at a set of ordered pairs and determine if the inverse relation will be a function?

(10) How can you look at an equation and determine if the inverse relation will be a function?

(11) How can you look at a graph and determine if the inverse relation will be a function?

(12) How can you look at a verbal statement and determine if the inverse verbal relationship will be a function?

**Defining Inverse Functions**

(13) Complete the function machine for \( f(g(x)) \) and \( g(f(x)) \) using the functions \( f(x) = 2x + 4 \) and \( g(x) = \frac{1}{2} x - 2 \) to find \( f(g(2)) \) and \( g(f(2)) \).

    **Solution:**

    \[
    \begin{align*}
    x &= 2 & f(x) &= 2x + 4 & g(x) &= \frac{1}{2} x - 2 \\
    x &= 2 & g(x) &= \frac{1}{2} x - 2 & f(x) &= 2x + 4 \\
    f(g(2)) &= & g(f(2)) &= 
    \end{align*}
    \]
An Inverse function is defined in the following ways:
- Symbolically: If \( f \) is a function, then \( f^{-1}(x) \) is the inverse function if \( f(f^{-1}(x)) = f^{-1}(f(x)) = x \). (Note: When the \(-1\) exponent is on the \( f \), it means inverse function and does not mean reciprocal.)
- Verbally: When you compose a function with its inverse and the inverse with its function, the resulting rule is the identity function \( y = x \).
- Numerically: The domain of a function is the range of the inverse function and vice versa.
- Graphically: A function and its inverse are symmetric to each other over the line \( y = x \).

Finding Equation for Inverse Functions: Find the inverse of the following functions by replacing the \( f(x) \) with \( y \), interchanging \( x \) and \( y \), and solving for \( y \). Rename \( y \) as \( f^{-1}(x) \), graph \( f(x) \) and \( f^{-1}(x) \) on the same graph, and find the domain and range of both.

(14) \( f(x) = 3x + 4 \)

\[ f^{-1}(x) = \frac{x - 4}{3} \]

Domain of \( f(x) \): \( \mathbb{R} \)

Range of \( f(x) \): \( \mathbb{R} \)

Domain of \( f^{-1}(x) \): \( \mathbb{R} \)

Range of \( f^{-1}(x) \): \( \mathbb{R} \)

(15) \( f(x) = 2|x - 1| \) on the domain \( x \leq 1 \)

\[ f^{-1}(x) = \frac{x}{2} \]

Domain of \( f(x) \): \( [0, 1] \)

Range of \( f(x) \): \( [0, 1] \)

Domain of \( f^{-1}(x) \): \( \mathbb{R} \)

Range of \( f^{-1}(x) \): \( \mathbb{R} \)
Recognizing Inverse Functions

An inverse relation is defined as any relation that swaps the independent and dependent variables. Determine whether the inverse of every function is a function by swapping the variables in each relation below to determine if the new relation is a function, and why or why not.

1. The set of ordered pairs \( \{(x, y) : (1, 2), (3, 5), (3, 6), (7, 5), (8, 2)\} \)
   Swap the \( x \) and \( y \) and write a new set of ordered pairs: \( \{(y, x) : (2, 1), (5, 3), (6, 3), (5, 7), (2, 8)\} \)
   Is the new relation a function of \( x \)? ___ no ___ Explain why or why not. ________________

   The number 2 maps onto 1 and 8

2. The set of ordered pairs \( \{(s, d) : (1, 1), (2, 4), (3, 9), (-1, 1), (-2, 4), (-3, 9)\} \)
   Swap the \( s \) and \( d \) and write a new set of ordered pairs: \( \{(d, s) : (1, 1), (4, 2), (9, 3), (1, -1), (4, -2), (9, -3)\} \)
   Is the new relation a function of \( s \)? ___ no ___ Explain why or why not. ________________

   Dependent variables are repeated

3. The relationship “\( x \) is a student of \( y \)”
   Write the words for the inverse relationship. “\( y \) is a teacher of \( x \)”
   Is the new relation a function of \( x \)? ___ no ___ Explain why or why not. ________________

   A teacher can have more than one student

4. The relationship “\( x \) is the biological daughter of mother \( y \)”
   Write the words for the inverse relationship. “\( y \) is the biological mother of daughter \( x \)”
   Is the new relation a function of \( x \)? ___ no ___ Explain why or why not. ________________

   A mother can have many daughters

5. The equation \( 2x + 3y = 6 \)
   Swap the \( x \) and \( y \) and write a new equation: \( 2y + 3x = 6 \)
   Is the new relation a function of \( x \)? ___ Yes ___ Explain why or why not. ________________

   For every \( x \) there is only one \( y \)

6. The equation \( x + y^2 = 9 \)
   Swap the \( x \) and \( y \) and write a new equation: \( y + x^2 = 9 \)
   Is the new relation a function of \( x \)? ___ Yes ___ Explain why or why not. ________________

   For every \( x \) there is only one \( y \)
(7) the equation \( y = x^2 + 4 \)
Swap the \( x \) and \( y \) and write a new equation: \( x = y^2 + 9 \)
Is the new relation a function of \( x? \) \text{no} \ Explain why or why not.
\( \text{two } y \text{ values for each } x \)

(8) the graph of the circle

If you swapped the \( x \) and \( y \) what would the new graph look like? \text{same as the old relation} \ Explain why or why not.
\( \text{two } y \text{ values for each } x \)

Answer the following questions:

(9) How can you look at a set of ordered pairs and determine if the inverse relation will be a function?

\text{Looking at ordered pairs, neither the independent nor the dependent variables can be repeated}

(10) How can you look at an equation and determine if the inverse relation will be a function?

\text{neither the } x \text{ nor the } y \text{ can be raised to an even power}

(11) How can you look at a graph and determine if the inverse relation will be functions?

\text{neither a vertical nor a horizontal line can intersect the graph at two points.}

(12) How can you look at a verbal statement and determine if the inverse verbal relationship will be a function?

\text{neither the independent not the dependent variable may be repeated}

\textbf{Defining Inverse Functions}

(13) Complete the function machine for \( f(g(x)) \) and \( g(f(x)) \) using the functions \( f(x) = 2x + 4 \) and \( g(x) = \frac{1}{2}x - 2 \) to find \( f(g(2)) \) and \( g(f(2)) \).

\[
\begin{array}{cccc}
  x = 2 & f(x) = 2x+4 & 8 & g(x) = \frac{1}{2}x - 2 & 2 \\
  x = 2 & g(x) = \frac{1}{2}x - 2 & -1 & f(x) = 2x+4 & 2 \\
\end{array}
\]

\( f(g(2)) = \boxed{2} \quad g(f(2)) = \boxed{2} \)
Inverse function is defined in the following ways:

- **Symbolically:** If \( f \) is a function, then \( f^{-1}(x) \) is the inverse function if
  \[ f(f^{-1}(x)) = f^{-1}(f(x)) = x. \]
  (Note: When the \(-1\) exponent is on the \( f \), it means inverse function and does not mean reciprocal.)

- **Verbally:** When you compose a function with its inverse and the inverse with its function, the resulting rule is the identity function \( y = x \).

- **Numerically:** The domain of a function is the range of the inverse function and vice versa.

- **Graphically:** A function and its inverse are symmetric to each other over the line \( y = x \).

Finding Equation for Inverse Functions: Find the inverse of the following functions by replacing the \( f(x) \) with \( y \), interchanging \( x \) and \( y \), and solving for \( y \). Rename \( y \) as \( f^{-1}(x) \), graph \( f(x) \) and \( f^{-1}(x) \) on the same graph, and find the domain and range of both.

14. \( f(x) = 3x + 4 \)

\[ f^{-1}(x) = \frac{1}{3}x - \frac{4}{3} \]

- Domain of \( f(x) \): all reals
- Range of \( f(x) \): all reals
- Domain of \( f^{-1}(x) \): all reals
- Range of \( f^{-1}(x) \): all reals

15. \( f(x) = 2|x - 1| \) on the domain \( x \leq 1 \)

\[ f^{-1}(x) = -x + 2 \text{ on the domain } x > 0 \]

- Domain of \( f(x) \): \((-\infty, 1]\)
- Range of \( f(x) \): \([0, \infty)\)
- Domain of \( f^{-1}(x) \): \([0, \infty)\)
- Range of \( f^{-1}(x) \): \((-\infty, 1]\)
Unit 2, Ongoing Activity, Little Black Book of Algebra II Properties

Little Black Book of Algebra II Properties
Unit 2 - Polynomial Equations & Inequalities

2.1 **Laws of Exponents** - record the rules for adding, subtracting, multiplying and dividing quantities containing exponents, raising an exponent to a power, and using zero and negative values for exponents.

2.2 **Polynomial Terminology** – define and write examples of monomials, binomials, trinomials, polynomials, the degree of a polynomial, a leading coefficient, a quadratic trinomial, a quadratic term, a linear term, a constant, and a prime polynomial.

2.3 **Special Binomial Products** – define and give examples of perfect square trinomials and conjugates, write the formulas and the verbal rules for expanding the special products \((a+b)^2, (a-b)^2, (a+b)(a-b)\), and explain the meaning of the acronym FOIL.

2.4 **Binomial Expansion using Pascal’s Triangle** – create Pascal’s triangle through row 7, describe how to make it, explain the triangle’s use in binomial expansion, and use the process to expand both \((a + b)^5\) and \((a – b)^5\).

2.5 **Common Factoring Patterns** - define and give examples of factoring using the greatest common factor of the terms, the difference in two perfect squares, the sum/difference in two perfect cubes, the square of a sum/difference \((a^2 + 2ab + b^2, a^2 – 2ab + b^2)\), and the technique of grouping.

2.6 **Zero–Product Property** – explain the zero–product property and its relevance to factoring: Why is there a zero–product property and not a property like it for other numbers?

2.7 **Solving Polynomial Equations** – identify the steps in solving polynomial equations, define double root, triple root, and multiplicity, and provide one reason for the prohibition of dividing both sides of an equation by a variable.

2.8 **Introduction to Graphs of Polynomial Functions** – explain the difference between roots and zeroes, define end behavior of a function, indicate the effect of the degree of the polynomial on its graph, explain the effect of the sign of the leading coefficient on the graph of a polynomial, and describe the effect of even and odd multiplicity on a graph.

2.9 **Polynomial Regression Equations** – explain the Method of Finite Differences to determine the degree of the polynomial that is represented by data.

2.10 **Solving Polynomial Inequalities** – indicate various ways of solving polynomial inequalities such as using the sign chart and using the graph. Provide two reasons for the prohibition against dividing both sides of an inequality by a variable.
Unit 2, Activity 2, Expanding Binomials Discovery Worksheet

Name_________________________________________ Date________________________

Expanding Binomials

Pascal's Triangle is an arithmetical triangle that can be used for some neat things in mathematics. Here's how to construct it:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

(1) Find a pattern and write a rule to develop Pascal’s triangle, then complete the next row._____

(2) Compare Pascal’s Triangle to the expansions of the Bellringer problems. Determine which row is used in which expansion.

(3) Pascal’s triangle only supplies the coefficients. Explain how to determine the exponents.

(4) Expand each of the following by hand:
   a. \((a - b)^2\) _____________________________________________________
   b. \((a - b)^3\) _____________________________________________________
   c. How can the rule for a sum be modified to use with a difference?

(5) Expand each of the following using Pascal’s triangle and then simplify each.
   a. \((a + b)^6\)
   b. \((a - b)^7\)
   c. \((2x + 3y)^4\)
Using Combinations to Expand Binomials

(1) How many subset combinations will there be of \{3, 5, 9\} if taken two at a time?

(2) Reviewing combinations learned in Algebra I, the symbol \(nC_r\) and \(\binom{n}{r}\) mean the combination of \(n\) things taken \(r\) at a time. Write problem #1 using these symbols. ________________

(3) Use the set \{a, b, c, d\} and list the sets which represent each of the following:
   a. 4 elements taken 1 at a time or \(4C_1\)
   b. 4 elements taken 2 at a time or \(4C_2\)
   c. 4 elements taken 3 at a time or \(4C_3\)
   d. 4 elements taken 4 at a time or \(4C_4\)

(4) What is the relationship between Pascal’s triangle and combinations?

(5) Explain two ways that \(nC_r\) is used in this lesson?

(6) Calculator Activity: Locate the \(nC_r\) button on the graphing calculator and use it to check your last row on Pascal’s triangle. Enter \(y = 7 \times nC_r \times x\) in the calculator. (m [PRB] 3:nCr on the TI–83/84 graphing calculator) Set the table to start at 0 with increments of 1. Create the table and compare the values to Pascal’s Triangle.
   Use this feature to expand \((a + b)^9\)
Expanding Binomials

Pascal's Triangle is an arithmetical triangle that can be used for some neat things in mathematics. Here's how to construct it:

```
  1
1  1
1  2  1
1  3  3  1
1  4  6  4  1
1  5 10 10  5  1
1  6 15 20 15  6  1
```

1  7 21 35 35 21  7  1

(1) Find a pattern and write a rule to develop Pascal’s triangle, then complete the next row._____

_The row always starts with 1. Then you add the two numbers above it. The numbers are also symmetric on both sides._

(2) Compare Pascal’s Triangle to the expansions of the Bellringer problems. Determine which row is used in which expansion.

_Each row is the coefficients of the terms. The power of the binomial is the 2nd term of the row._

(3) Pascal’s triangle only supplies the coefficients. Explain how to determine the exponents.

_The exponents of the first term start at the power of the binomial and decrease by one each time until 0. The exponents of the 2nd term start at 0 and increase by one until the power of the binomial. The sum of the exponents of a and b is the power of the binomial._

(4) Expand each of the following by hand:

- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- How can the rule for a sum be modified to use with a difference?

_The signs start with + then alternate._

(5) Expand each of the following using Pascal’s triangle and then simplify each.

- $(a + b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$
- $(a - b)^7 = a^7 - 7a^6b + 21a^5b^2 - 35a^4b^3 + 35a^3b^4 - 21a^2b^5 + 7ab^6 - b^7$
- $(2x + 3y)^4 = 16x^4 + 24x^3y + 36x^2y^2 + 54xy^3 + 81y^4$
Using Combinations to Expand Binomials

(1) How many subset combinations will there be of \{3, 5, 9\} if taken two at a time?

three – \{3, 5\}, \{3, 9\}, \{5, 9\}

(2) Reviewing combinations learned in Algebra I, the symbol \(\binom{n}{r}\) mean the combination

of \(n\) things taken \(r\) at a time. Write problem #1 using these symbols.

\(3C_2\) or \(\binom{3}{2}\)

(3) Use the set \{a, b, c, d\} and list the sets which represent each of the following:

a. 4 elements taken 1 at a time or \(4C_1\) \{a\}, \{b\}, \{c\}, \{d\}

b. 4 elements taken 2 at a time or \(4C_2\) \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}

c. 4 elements taken 3 at a time or \(4C_3\) \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}

d. 4 elements taken 4 at a time or \(4C_4\) \{a, b, c, d\}

(4) What is the relationship between Pascal’s triangle and combinations?

The numbers of subsets in each are the numbers in the row of Pascal’s triangle, 1, 4, 6, 4, 1

(5) Explain two ways that \(nC_r\) is used in this lesson?

It can be used to find the combination of terms in a set or to find the coefficients used in binomial expansion.

(6) Calculator Activity: Locate the \(nC_r\) button on the graphing calculator and use it to check your last row on Pascal’s triangle. Enter \(y = 7 \cdot nC_r \cdot x\) in the calculator. (m \[PRB\] 3:nCr on the TI–83/84 graphing calculator) Set the table to start at 0 with increments of 1. Create the table and compare the values to Pascal’s Triangle.

Use this feature to expand \((a + b)^9\)

\(a^9 + 9a^8b + 36a^7b^2 + 84a^6b^3 + 126a^5b^4 + 126a^4b^5 + 84a^3b^6 + 36a^2b^7 + 9ab^8 + b^9\)
Investigating Graphs of Polynomials

Using a graphing calculator, graph each of the equations on the same graph in the specified window. Find the zeroes and sketch the graph locating zeroes and answer the questions.

(1) \( y_1 = x^2 + 7x + 10 \)
\( y_2 = 3x^2 + 21x + 30, \)
\( y_3 = \frac{1}{2} x^2 + \frac{7}{2} x + \frac{10}{2}. \)

(a) Write the equations in complete factored form and check graphs to see if both forms are equivalent.
\( y_1 = \) ________________
\( y_2 = \) ________________
\( y_3 = \) ________________

(b) List the zeroes: ________________

(c) How many zeroes? ________________
(d) How many roots? ________________

(e) What is the effect of the constant factor on the zeroes? ________________

(f) What is the effect of the constant factor on the shape of the graph? ________________

(g) Discuss end behavior.

(h) Graph \( y_4 = 3y_1 \) (On calculator, find \( y_1 \) under \( Y-VARS, 1: \) Function, \( 1: Y_1. \)) Looking at the graphs, which other equation is this equivalent to? _____

(i) This transformation is in the form \( kf(x) \). How does \( k > 0 \) affects the graph? ________________
Unit 2, Activity 7, Graphing Polynomials Discovery Worksheet

(2) \( y_1 = x^3 + 6x^2 + 8x \)
\( y_2 = -x^3 - 6x^2 - 8x \).

(a) Write the equations in complete factored form and check graphs to see if both forms are equivalent.
\[ y_1 = \ldots \]
\[ y_2 = \ldots \]

(b) List the zeroes: ______
(c) How many zeroes? _____
(d) How many roots? _____

(e) What effect does a common factor of \(-1\) have on the zeroes? ________________

(f) What is the effect of a common factor of \(-1\) on the end behavior? ________________

(g) Since this transformation is in the form \(kf(x)\), how does \(k = -1\) affect the graph? ________________

(3) \( y_1 = x^2 - 6x + 9 \)
\( y_2 = x^2 + 4x + 4 \).

(a) Write the equations in complete factored form and check graphs to see if both forms are equivalent.
\[ y_1 = \ldots \]
\[ y_2 = \ldots \]

(b) List the zeroes: \( y_1: \ldots \) \( y_2: \ldots \)
(c) How many zeroes in each? ________________
(d) How many roots? _____
(e) Discuss multiplicity. ________________

(f) What does the graph look like when there is a double root? ________________
(4) \( y_1 = x^3 - x^2 - 8x + 12 \)

(a) List the zeroes: ______________________

(b) Use the zeroes to write the equation in factored form and check graphs to see if both forms are equivalent.

\( y_1 = \) ______________________

c) How many zeroes? ______________________

d) How many roots? ______________________

e) Discuss multiplicity and its effect on the graph.

______________________________

(f) Discuss end behavior. ______________________

(5) \( y_1 = x^4 - 3x^3 - 10x^2 + 24x \)

(a) List the zeroes: ______________________

(b) Use the zeroes to write the equation in factored form and check graphs to see if both forms are equivalent.

\( y_1 = \) ______________________

c) How many zeroes? ______________________

d) How many roots? ______________________

e) Looking at the number of roots in problems 1 through 4, how can you determine how many roots a polynomial has? ______________________

(f) Discuss end behavior. ______________________

g) Graph \( y_2 = -y_1 \). What is the effect on the zeroes and the end behavior? ______________________

(h) Looking at the end-behavior in problems 1 through 4, how can you predict end behavior?

______________________________
(6) \( y_1 = x^4 + 2x^3 - 11x^2 - 12x + 36 \)

(a) List the zeroes: ________________________________

(b) Use the zeroes to write the equation in factored form and check graphs to see if both forms are equivalent.

\[ y_1 = \] ________________________________

(c) How many zeroes? __________________

(d) How many roots? __________________

(e) Discuss multiplicity.______________________________

(f) Discuss end behavior. ________________________________

(7) \( y_1 = x^5 - 6x^4 + 9x^3 \)

(a) List the zeroes: ________________________________

(b) Use the zeroes to write the equation in factored form and check graphs to see if both forms are equivalent.

\[ y_1 = \] ________________________________

(c) How many zeroes? _____ (d) How many roots? ___

(e) Discuss multiplicity.______________________________

(f) Discuss end behavior. ________________________________

(g) What is the difference in the looks of the graph for a double root and a triple root?_______

______________________________________________
Investigating Graphs of Polynomials

Using a graphing calculator, graph each of the equations on the same graph in the specified window. Find the zeroes and sketch the graph locating zeroes and answer the questions.

(1) \( y_1 = x^2 + 7x + 10 \)
\( y_2 = 3x^2 + 21x + 30, \)
\( y_3 = \frac{1}{2}x^2 + \frac{7}{2}x + \frac{10}{2}. \)

(a) Write the equations in complete factored form and check graphs to see if both forms are equivalent.
\( y_1 = (x + 5)(x + 2) \)
\( y_2 = 3(x + 5)(x + 2) \)
\( y_3 = \frac{1}{2}(x + 5)(x + 2) \)

(b) List the zeroes: \{-2, -5\}

(c) How many zeroes? 2
(d) How many roots? 2

(e) What is the effect of the constant factor on the zeroes? nothing

(f) What is the effect of the constant factor on the shape of the graph? if constant is > 1 stretched vertically, more steep between zeroes, < 1 wider

(g) Discuss end behavior. starts up and ends up

(h) Graph \( y_4 = 3y_1 \) (On calculator, find \( y_1 \) under v, Y-VARS, 1: Function, 1: Y1. ) Looking at the graphs, which other equation is this equivalent to? \( y_3 \)

(i) This transformation is in the form \( kf(x) \). How does \( k > 0 \) affects the graph? \( k > 0 \) does not affect the zeroes but does stretch the graph vertically affecting the \( y \)-values or the range
(2) \( y_1 = x^3 + 6x^2 + 8x \)
\[ y_2 = -x^3 - 6x^2 - 8x \]

(a) Write the equations in complete factored form and check graphs to see if both forms are equivalent.

\[ y_1 = x(x + 4)(x + 2) \]
\[ y_2 = -1 \cdot x(x + 4)(x + 2) \]

(b) List the zeroes: \{-4, -2, 0\} (c) How many zeroes? 3 (d) How many roots? 3

(e) What effect does a common factor of \(-1\) have on the zeroes? nothing

(f) What is the effect of a common factor of \(-1\) on the end behavior? rotates it on the \(x\)-axis so when \(y_1\) started down and ended up, the negative made it start up and end down.

(g) Since this transformation is in the form \(kf(x)\), how does \(k = -1\) affect the graph? rotates it on the \(x\)-axis, all positive \(y\) values are now negative and all negative \(y\) values are now positive

(3) \( y_1 = x^2 - 6x + 9 \)
\[ y_2 = x^2 + 4x + 4 \]

(a) Write the equations in complete factored form and check graphs to see if both forms are equivalent.

\[ y_1 = (x - 3)^2 \]
\[ y_2 = (x + 2)^2 \]

(b) List the zeroes: \(y_1: \{3\}\) \(y_2: \{-2\}\) (c) How many zeroes in each? one

d How many roots? two (e) Discuss multiplicity. There is one double root so we say there the root has a multiplicity of two – one zero, two roots

(f) What does the graph look like when there is a double root? it skims off the \(x\)-axis
(4) $y_1 = x^3 - x^2 - 8x + 12$

(a) List the zeroes: $\{-3, 2\}$

(b) Use the zeroes to write the equation in factored form and check graphs to see if both forms are equivalent.

\[ y_1 = (x + 3)(x - 2)^2 \]

(c) How many zeroes? 2

(d) How many roots? 3

(e) Discuss multiplicity and its effect on the graph. There is a single root and a double root. The double root is the location where the graph skims off the x-axis.

(f) Discuss end behavior. Starts down and ends up

(5) $y_1 = x^4 - 3x^3 - 10x^2 + 24x$

(a) List the zeroes: $\{-3, 0, 2, 4\}$

(b) Use the zeroes to write the equation in factored form and check graphs to see if both forms are equivalent.

\[ y_1 = x(x + 3)(x - 2)(x - 4) \]

(c) How many zeroes? 4

(d) How many roots? 4

(e) Looking at the number of roots in problems 1 through 4, how can you determine how many roots a polynomial has? Highest exponent

(f) Discuss end behavior. Starts up and ends up

(g) Graph $y_2 = -y_1$. What is the effect on the zeroes and the end behavior? Opposite

(h) Looking at the end-behavior in problems 1 through 4, how can you predict end behavior? Highest exponent odd and positive – starts down ends up, odd and negative, starts up and ends down, even and positive – starts up and ends up, even and negative – starts down and ends down
(6) \( y_1 = x^4 + 2x^3 - 11x^2 - 12x + 36 \)

(a) List the zeroes: \{-3, 2\}

(b) Use the zeroes to write the equation in factored form and check graphs to see if both forms are equivalent.

\[ y_1 = (x + 3)^2 (x - 2)^2 \]

(c) How many zeroes? 2

(d) How many roots? 4

(e) Discuss multiplicity. 2 sets of double roots

(f) Discuss end behavior. starts up and ends up

(7) \( y_1 = x^5 - 6x^4 + 9x^3 \)

(a) List the zeroes: \{0, 3\}

(b) Use the zeroes to write the equation in factored form and check graphs to see if both forms are equivalent.

\[ y_1 = x^3(x - 3)^2 \]

(c) How many zeroes? 2

(d) How many roots? 5

(e) Discuss multiplicity. one double root and one triple root

(f) Discuss end behavior. starts down and ends up

(g) What is the difference in the looks of the graph for a double root and a triple root? a double root or any root created by an even exponent skims off the x-axis while a triple root or any root created by an odd exponent flattens out and goes through the x-axis
Predicting Degree of Polynomial by Zeroes

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-36</td>
<td>0</td>
<td>8</td>
<td>0</td>
<td>-12</td>
<td>-16</td>
<td>0</td>
<td>48</td>
</tr>
</tbody>
</table>

(1) Using the data above, what would be the least degree of a polynomial that would model the data and explain why.

(2) Predict the equation of the polynomial in factored form.

(3) Plot the data in a graphing calculator and make a scatter plot. (To enter data on a TI-84 calculator: STAT, 1:Edit, enter data into L1 and L2. To set up the plot of the data: 2nd, STAT PLOT, 1:PLOT1, ENTER, On, Type: à, Xlist: L1, Ylist: L2, Mark (any). To graph the scatter plot: ZOOM, 9: ZoomStat.) Then enter the equation to see if it matches the data. Adjust the leading coefficient of the equation until the graph matches the data and write the final equation.

Method of Finite Differences

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-14</td>
<td>-11</td>
<td>-8</td>
<td>-5</td>
<td>-2</td>
<td>1</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

(4) Using the data above, what would be the least degree of a polynomial that would model the data and explain why you came to that conclusion.

(5) Use the Method of Finite Differences to twice subtract the y-values to get 0 to determine that the function is linear, a polynomial of first degree. Find the equation of the line.

(6) Apply the method of finite differences to the first table several times and develop the guidelines for determining the degree of the polynomial:

\[ y = c \] if ________________

\[ y = ax + b \] if ________________

\[ y = ax^2 + bx + c \] if ________________

\[ y = ax^3 + bx^2 + cx + d \] if ________________

(7) What are the limitations of using this method in evaluating real-life data?

Real Life Application

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
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<th></th>
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<tr>
<td>number</td>
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<td>56</td>
<td>65</td>
<td>75</td>
<td>94</td>
<td>110</td>
</tr>
</tbody>
</table>

Because of improved health care, people are living longer. The above data relates the number of Americans (in thousands) who are expected to be over 100 years old for the selected years. (Source: US Census Bureau) Enter the data into the calculator, letting \( x = 4 \) correspond to 1994 and make a scatter plot. Then graph the following equations:

\[ y_1 = 6.057x + 20.4857, \ y_2 = 0.4018x^2 – 1.175x + 48.343, \ y_3 = -0.007x^3 +0.5893x^2 – 2.722 + 52.1428. \]

- Which polynomial best models the number of Americans over 100 years old? ________________
- Use the equation chosen to predict the number of Americans who will be over 100 years old in the year 2008. ________________
Predicting Degree of Polynomial by Zeroes

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
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<td>-16</td>
<td>0</td>
<td>48</td>
</tr>
</tbody>
</table>

(1) Using the data above, what would be the least degree of a polynomial that would model the data and explain why.
3rd degree polynomial because there are three zeroes

(2) Predict the equation of the polynomial in factored form. 
\[ y = x(x - 3)(x + 2) \]

(3) Plot the data in a graphing calculator and make a scatter plot. (To enter data on a TI-84 calculator:
STAT, 1:Edit, enter data into L1 and L2. To set up the plot of the data: 2nd, STAT PLOT, 1:PLOT1, ENTER, On, Type: à, Xlist: L1, Ylist: L2, Mark (any). To graph the scatter plot: ZOOM, 9: ZoomStat.) Then enter the equation to see if it matches the data. Adjust the leading coefficient of the equation until the graph matches the data and write the final equation. 
\[ f(x) = 2x(x - 3)(x + 2) \]

Method of Finite Differences

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<tr>
<th>x</th>
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<td>-11</td>
<td>-8</td>
<td>-5</td>
<td>-2</td>
<td>1</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

(4) Using the data above, what would be the least degree of a polynomial that would model the data and explain why you came to that conclusion.
1st degree because the change in y over the change in x (slope) is constant

(5) Use the Method of Finite Differences to twice subtract the y-values to get 0 to determine that the function is linear, a polynomial of first degree. Find the equation of the line. 
\[ f(x) = 3x - 5 \]

(6) Apply the method of finite differences to the first table several times and develop the guidelines for determining the degree of the polynomial:
- \( y = c \) if the 1st order differences are 0
- \( y = ax + b \) if the 2nd order differences are 0
- \( y = ax^2 + bx + c \) if the 3rd order differences are 0
- \( y = ax^3 + bx^2 + cx + d \) if the 4th order differences are 0

(7) What are the limitations of using this method in evaluating real-life data?
Real-world data is not exact

Real Life Application

<table>
<thead>
<tr>
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<tr>
<td>number</td>
<td>50</td>
<td>56</td>
<td>65</td>
<td>75</td>
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</tbody>
</table>

Because of improved health care, people are living longer. The above data relates the number of Americans (in thousands) who are expected to be over 100 years old for the selected years. (Source: US Census Bureau) Enter the data into the calculator, letting \( x = 4 \) correspond to 1994 and make a scatter plot. Then graph the following equations:
\[ y_1 = 6.057x + 20.4857, \quad y_2 = 0.4018x^2 - 1.175x + 48.343, \quad y_3 = -0.007x^3 +0.5893x^2 - 2.722 + 52.1428 \]

- Which polynomial best models the number of Americans over 100 years old: \( y_3 \)
- Use the equation chosen to predict the number of Americans who will be over 100 years old in the year 2008. \( y_3(18) = 153.571 \) Americans
Unit 2, Activity 9, Solving Polynomial Inequalities by Graphing

Name_________________________________ Date_____________________

**Polynomial Inequalities** The equations in the Bellringer have only one variable. However, it is helpful to use a two-variable graph to quickly solve a one-variable inequality. Fast graph the following polynomial functions only paying attention to the x–intercepts and the end-behavior. Use the graphs to solve the one-variable inequalities by looking at the positive and negative values of y.

(1) Graph \( y = -2x + 6 \)

Solve for \( x: -2x + 6 > 0 \)

(2) Graph \( y = x(x - 4) \)

Solve for \( x: x(x - 4) > 0 \)

(3) Graph \( y = x(x - 4) \)

Solve for \( x: x(x - 4) \leq 0 \)

(4) Graph \( y = (x - 3)(x + 4)(x - 7) \)

Solve for \( x: (x - 3)(x + 4)(x - 7) \geq 0 \)

(5) Graph \( y = x^2 - 9x + 14 \)

Solve for \( x: x^2 - 9x < -14 \)

(6) Graph \( y = 5x^3 - 15x^2 \)

Solve for \( x: 5x^3 \leq 15x^2 \) *(Hint: Isolate 0 first.)*
Unit 2, Activity 9, Solving Polynomial Inequalities by Graphing with Answers

Name_________________________________________ Date____________________

**Polynomial Inequalities**  The equations in the Bellringer have only one variable. However, it is helpful to use a two-variable graph to quickly solve a one-variable inequality. Fast graph the following polynomial functions only paying attention to the x–intercepts and the end-behavior. Use the graphs to solve the one-variable inequalities by looking at the positive and negative values of y.

(1) Graph \( y = -2x + 6 \)

![Graph of \( y = -2x + 6 \)]

Solve for \( x \): \(-2x + 6 > 0\)  
\( (-\infty, 3) \)

(2) Graph \( y = x(x - 4) \)

![Graph of \( y = x(x - 4) \)]

Solve for \( x \): \( x(x - 4) > 0 \)  
\( (-\infty, 0) \cup (4, \infty) \)

(3) Graph \( y = x(x - 4) \)

![Graph of \( y = x(x - 4) \)]

Solve for \( x \): \( x(x - 4) \leq 0 \)  
\( [0, 4] \)

(4) Graph \( y = (x - 3)(x + 4)(x - 7) \)

![Graph of \( y = (x - 3)(x + 4)(x - 7) \)]

Solve for \( x \): \((x - 3)(x + 4)(x - 7) \geq 0\)  
\( [-4, 3] \cup [7, \infty) \)

(5) Graph \( y = x^2 - 9x + 14 \)

![Graph of \( y = x^2 - 9x + 14 \)]

Solve for \( x \): \( x^2 - 9x < -14 \) (Hint: Isolate 0 first.)  
\( (2, 9) \)

(6) Graph \( y = 5x^3 - 15x^2 \)

![Graph of \( y = 5x^3 - 15x^2 \)]

Solve for \( x \): \( 5x^3 \leq 15x^2 \) (Hint: Isolate 0 first.)  
\( (-\infty, 3) \)
Unit 2, Activity 7, Specific Assessment Graphing Polynomials

Graphs of Polynomials
Find the zeroes and use the rules developed in the Graphing Polynomials Discovery Worksheet to sketch the following graphs without a calculator. Label accurately the zeroes, end–behavior, and $y$–intercepts. Do not be concerned with minimum and maximum values between zeroes.

a. $y = x^3 - 8x^2 + 16x$

b. $y = -2x^2 - 12x - 10$

c. $y = (x - 4)(x + 3)(x + 1)$

d. $y = -(x + 2)(x - 7)(x + 5)$

e. $y = (2 - x)(3 - x)(5 + x)$

f. $y = x^2 + 10 + 25$

g. $y = (x - 3)^2(x + 5)$

h. $y = (x - 3)^3(x + 5)$

i. $y = (x - 3)^3(x + 5)^2$

j. $y = (x - 3)^4(x + 5)$

Blackline Masters, Algebra II
Louisiana Comprehensive Curriculum, Revised 2008
**Unit 2, Activity 7, Specific Assessment Graphing Polynomials with Answers**

Find the zeroes and use the rules you developed in the Discovery Worksheet to sketch the following graphs without a calculator. The following must be labeled and accurate: zeroes, end behavior, and y intercepts. Do not be concerned with minimum and maximum values between zeroes.

a. $y = x^3 - 8x^2 + 16x$

b. $y = -2x^2 - 12x - 10$

c. $y = (x - 4)(x + 3)(x + 1)$

d. $y = -(x + 2)(x - 7)(x + 5)$

e. $y = (2 - x)(3 - x)(5 + x)$

f. $y = x^2 + 10 + 25$

g. $y = (x - 3)^2(x + 5)$

h. $y = (x - 3)^3(x + 5)$

i. $y = (x - 3)^3(x + 5)^2$

j. $y = (x - 3)^4(x + 5)$
3-1 **Rational Terminology** – define rational number, rational expression, and rational function, least common denominator (LCD), complex rational expression.
3-2 **Rational Expressions** – explain the process for simplifying, adding, subtracting, multiplying, and dividing rational expressions; define reciprocal, and explain how to find denominator restrictions.
3-3 **Complex Rational Expressions** – define and explain how to simplify.
3-4 **Vertical Asymptotes of Rational Functions** – explain how to find domain restrictions and what the domain restrictions look like on a graph, explain how to determine end-behavior of a rational function around a vertical asymptote.
3-5 **Solving Rational Equations** – explain the difference between a rational expression and a rational equation, list two ways to solve rational equations, define extraneous roots.
3-6 **Solving Rational Inequalities** – list the steps for solving an inequality by using the sign chart method.
Algebra II – Date

Simplify

(1) \((x^2)(x^5)\)

(2) \((x^2y^5)^4\)

(3) \(\frac{(x^5)^3}{x^8}\)

(4) \(\frac{x^7}{x^7}\)

(5) \(\frac{x^3}{x^5}\)

(6) Choose one problem above and write in a sentence the Law of Exponents used to determine the solution.
Unit 3, Activity 1, Simplifying Rational Expressions

Name_________________________________________ Date________________________

Laws of Exponents

I. Enter the following in your calculators on the home screen:
(1) $3^0 = \underline{1}$ (2) $2^{-3}$ and $\frac{1}{2^3} = \underline{\frac{1}{8}}$ (3) $0.001$ and $\frac{1}{10^3} = \underline{0.001}$ (4) $0.0037$ and $3.7 \times 10^{-4} = \underline{0.00037}$

II. Simplify and write answers with only positive exponents.
(1) $(x^{-3})(x^4) = \underline{x} (2) (x^2)^3 = \underline{x^6} (3) \frac{x^3}{x^2} = \underline{x}$

III. Define rational number as the quotient of two integers $\frac{p}{q}$ in which $q \neq 0$ and define rational algebraic expression as the quotient of two polynomials $P(x)$, and $Q(x)$ in which $Q(x) \neq 0$. Find the denominator restrictions on the following rational expressions.
(1) $\frac{4x^3}{7t}$ (2) $\frac{3x + 5}{y - 3}$ (3) $\frac{2x + 5}{3x - 7}$ (4) $\frac{3x + 2}{x^2 - 5x + 6}$ (5) $\frac{4}{x^2 - 9}$

IV. Simplify $\frac{24}{40}$ and explain the steps you used.

V. Apply this concept to simplify the following expressions and develop the process to simplify rational expressions. Specify all denominator restrictions.
(Remember the denominator restrictions apply to the original problem, not the simplified problem.)
(1) $-\frac{27x^2y^4}{9x^4y} = \underline{-3y^{-2}x^{-2}}$ (2) $\frac{a-b}{b-a} = \underline{-1}$
(3) $\frac{8 - 2x}{x^2 - 5x + 6}$ (4) $8x(4x - 28)^{-1}$

VI. To verify that the denominator restrictions apply to the original problem, not the simplified problem, complete the following:
(1) Simplify $f(x) = \frac{x^3 - 2x^2 + 4x - 8}{x - 2}$
(2) Graph both the original and the simplified form on the graphing calculator. Trace to $x = 2$ on both to find $f(2)$ and $g(2)$. There is a hole in one graph and not in the other; therefore, they are only equal for all values of $x$ except $x = 2$. Verify this in a table: go to [2ND], [TBL SET], (above [WINDOW]) and TblStart = 0 and go up by increments ($\Delta Tbl$) = 0.2. Again you will see no value for $x = 2$.

Application
The side of a regular hexagon is $2ab^3$ and the side of a regular triangle is $3a^2b$. Find the ratio of the perimeter of the hexagon to the perimeter of the triangle. Show all your work:
Unit 3, Activity 1, Simplifying Rational Expressions with Answers

Laws of Exponents

I. Enter the following in your calculators on the home screen:
(1) \(3^0 = 1\)  
(2) \(2^{-3} = \frac{1}{8}\)  
(3) \(0.001 = 10^{-3}\)  
(4) \(0.00037 = 3.7 \times 10^{-4}\)

II. Simplify and write answers with only positive exponents.
(1) \((x^{-3})(x^4) = x\)  
(2) \((x^{-2})^3 = \frac{1}{x^6}\)  
(3) \(\frac{x^{-3}}{x^{-2}} = \frac{1}{x}\)

III. Define rational number as the quotient of two integers \(\frac{p}{q}\) in which \(q \neq 0\) and define rational algebraic expression as the quotient of two polynomials \(P(x), Q(x)\) in which \(Q(x) \neq 0\). Find the denominator restrictions on the following rational expressions.
(1) \(t\)  
(2) \(\frac{3x + 5}{y - 3}, y \neq 3\)  
(3) \(\frac{2x + 5}{3x - 7}, x \neq \frac{7}{3}\)  
(4) \(\frac{3x + 2}{x^2 - 5x + 6}, x \neq 3, x \neq 2\)  
(5) \(\frac{4}{x^2 - 9}, x \neq \pm 3\)

IV. Simplify \(\frac{24}{40}\) and explain the steps you used. \(\frac{24}{40} = \frac{3}{5} = \frac{6}{10}\), Use the identity element of multiplication.

V. Apply this concept to simplify the following expressions and develop the process to simplify rational expressions. Specify all denominator restrictions.
(1) \(-\frac{27x^2y^4}{9x^3y} = \frac{-3y^3}{x}, x \neq 0, y \neq 0\)
(2) \(\frac{a-b}{b-a} = -1, a \neq b\)
(3) \(\frac{8-2x}{x^2 - 5x + 6} = \frac{-4}{x - 3}, x \neq 2, x \neq 3\)
(4) \(8x(4x - 28)^{-1} = \frac{2x}{x - 7}, x \neq 7\)

VI. To verify that the denominator restrictions apply to the original problem, not the simplified problem, complete the following:
(1) Simplify \(f(x) = \frac{x^3 - 2x^2 + 4x - 8}{x - 2} = x^2 + 4, x \neq 2\)

(2) Graph both the original and the simplified form on the graphing calculator. Trace to \(x = 2\) on both to find \(f(2)\) and \(g(2)\). There is a hole in one graph and not in the other; therefore, they are only equal for all values of \(x\) except \(x = 2\). Verify this in a table: go to [2ND], [TBL SET], (above [WINDOW]) and TblStart = 0 and go up by increments (\(\Delta Tbl\) = 0.2). Again you will see no value for \(x = 2\).

Application

The side of a regular hexagon is \(2ab^3\) and the side of a regular triangle is \(3a^2b\). Find the ratio of the perimeter of the hexagon to the perimeter of the triangle. Show all your work: \(\frac{4b^2}{3a}\)
Adding/Subtracting Rational Expressions

I. State the Rule for Adding/Subtracting Rational Expressions: _______________________

II. Apply this process to find the sums:
   (1) \( \frac{x - 5}{6x^2} + \frac{2x + 6}{6x^2} \)
   (2) \( \frac{x - 5}{6x^2 - 54} + \frac{2x + 6}{x - 3} \)

III. Subtract and simplify.
   (3) \( \frac{2}{15} - \frac{7}{25} \)
   (4) \( \frac{2}{5} - 6 \)

   State the Rule: __________________________________________

IV. Apply this process to find the differences:
   (5) \( \frac{x - 5}{6x^2} - \frac{2x + 6}{6x^2} \)
   (6) \( \frac{x - 5}{6x^2 - 54} - \frac{2x + 6}{x - 3} \)

Application
The time it takes a boat to go downstream is represented by the function \( d(x) = \frac{2}{x + 1} \) hours, where \( x \) represents the number of miles. The time it takes a boat to go upstream is represented by the function \( u(x) = \frac{3}{x - 1} \) hours.

a. How long in minutes does it take to go 2 miles upstream? 2 miles downstream? Explain why it would be different?

b. Find a rational function \( f(x) \) for the total time in minutes. Then find the total time it takes to go a total of 2 miles upstream then back to the starting point.

c. Find a rational function \( g(x) \) for how much more time it takes to go upstream than downstream.

d. Find how much more time in minutes it takes to go upstream than downstream if you have traveled 2 miles upstream and back to the starting point.
Adding/Subtracting Rational Expressions

I. State the Rule for Adding/Subtracting Rational Expressions: **Find the LCD and add the numerators and keep the denominator.**

II. Apply this process to find the sums:

1. \[ \frac{x-5}{6x^2} + \frac{2x+6}{6x^2} = \frac{3x+1}{6x^2} \]
2. \[ \frac{x-5}{6x^2-54} + \frac{2x+6}{x-3} = \frac{12x^2 + 73x + 103}{6x^2 - 54} \]

III. Subtract and simplify.

3. \[ \frac{2}{15} - \frac{7}{25} = -\frac{11}{75} \]
4. \[ \frac{2}{5} - 6 = -\frac{28}{5} \]

State the Rule: **Find the LCD and subtract the numerators and keep the denominator.**

IV. Apply this process to find the differences:

5. \[ \frac{x-5}{6x^2} - \frac{2x+6}{6x^2} = -\frac{2x-11}{6x^2} \]
6. \[ \frac{x-5}{6x^2-54} - \frac{2x+6}{x-3} = -\frac{12x^2 - 71x - 113}{6x^2 - 54} \]

Application

The time it takes a boat to go downstream is represented by the function \( d(x) = \frac{2}{x + 1} \) hours, where \( x \) represents the number of miles. The time it takes a boat to go upstream is represented by the function \( u(x) = \frac{3}{x} \) hours.

a. How long in minutes does it take to go 2 miles upstream? 2 miles downstream? Explain why it would be different? \( u(2) = 3 \text{ hours} = 180 \text{ minutes}, d(2) = 40 \text{ minutes}, \text{current helps going downstream} \)
b. Find a rational function \( f(x) \) for the total time in minutes. Then find the total time it takes to go a total of 2 miles upstream then back to the starting point. \( u(x) + d(x) = f(x) = \frac{5x + 1}{x^2 - 1} \)

\( f(2) = 220 \text{ minutes} \)
c. Find a rational function \( g(x) \) for how much more time it takes to go upstream than downstream. \( u(x) - d(x) = g(x) = \frac{x + 5}{x^2 - 1} \)
d. Find how much more time in minutes it takes to go upstream than downstream if you have traveled 2 miles upstream and back to the starting point. \( g(2) = 140 \text{ minutes} \)
Unit 3, Activity 6, Rational Expressions Applications

Application Problems

1. John’s car uses 18 gallons to travel 300 miles. He has 7 gallons of gas in the car and wants to know how much more gas will be needed to drive 650 miles. Assuming the car continues to use gas at the same rate, how many more gallons will be needed? Set up a rational equation and solve.

2. What is the formula you learned in Algebra I concerning distance, rate, and time? Write a rational equation solved for time. Set up a rational equation and use it to solve the following problem: Jerry walks 6 miles per hour and travels for 5 miles. How many minutes does he walk?

3. Sue and Bob are walking down an airport concourse at the same speed. Bob jumps on a 600 foot moving sidewalk that travels 3 feet per second and ends at the airplane door. While on the sidewalk, he continues to walk at the same rate as Sue until he reaches the end. He beats Sue by 180 seconds. (a) Using the formula in #2, write the rational expression for Sue’s time. (b) Write the rational expression for Bob’s time. (c) Since Bob’s time is 180 seconds less that Sue’s time, write the rational equation that equates their times. (d) Solve for the walking rate.

4. List the 6-step process for solving application problems developed in Unit 1. (1) (2) (3) (4) (5) (6)

5. Remember the Algebra I formula: Amount of work \((A) = \text{rate} (r) \times \text{time} (t)\). Rewrite the equation as the rational equation isolating: . Mary plants flowers at a rate of 200 seeds per hour. How many seeds has she planted in 2 hours? Write the rational equation and answer in a sentence.

6. If one whole job can be accomplished in \(t\) units of time, then the rate of work is \(r = \frac{1}{t}\). Harry and Melanie are working on Lake Pontchartrain clean-up detail. (a) Harry can clean up the trash in his area in 6 hours. Write an equation for Harry’s rate. (b) Melanie can do the same job in 4 hours. Write an equation for Melanie’s rate. (c) If they work together, how long will it take them to clean that area? Write a rational equation for the job and solve.
Application Problems

1. John’s car uses 18 gallons to travel 300 miles. He has 7 gallons of gas in the car and wants to know how much more gas will be needed to drive 650 miles. Assuming the car continues to use gas at the same rate, how many more gallons will be needed? Set up a rational equation and solve.

\[
\frac{18 \text{ gal.}}{300 \text{ mi.}} = \frac{(7+x) \text{ gal.}}{650 \text{ mi.}}, \quad x = 32, \text{ John will need } 32 \text{ more gallons to drive 650 miles.}
\]

2. What is the formula you learned in Algebra I concerning distance, rate, and time? d = rt

Write a rational equation solved for time. \( t = \frac{d}{r} \)

Set up a rational equation and use it to solve the following problem: Jerry walks 6 miles per hour and travels for 5 miles. How many minutes does he walk? \( t = \frac{5 \text{ mi}}{6 \text{ mph}} \) or \( \frac{5}{6} \) of an hour, Jerry walks 50 minutes.

3. Sue and Bob are walking down an airport concourse at the same speed. Bob jumps on a 600 foot moving sidewalk that travels 3 feet per second and ends at the airplane door. While on the sidewalk, he continues to walk at the same rate as Sue until he reaches the end. He beats Sue by 180 seconds.

(a) Using the formula in #2, write the rational expression for Sue’s time.

\( \frac{600}{r} \)

(b) Write the rational expression for Bob’s time.

\( \frac{600}{r+3} \)

(c) Since Bob’s time is 180 seconds less that Sue’s time, write the rational equation that equates their times.

\( \frac{600}{r} - 180 = \frac{600}{r+3} \)

(d) Solve for the walking rate. \( r = 2 \), Sue and Bob are walking at a rate of 2 feet per second.

4. List the 6-step process for solving application problems developed in Unit 1.

1. Define the variables and the given information
2. Determine what you are asked to find
3. Write an equation
4. Solve the equation
5. Check
6. Answer the question in a sentence, include units

5. Remember the Algebra I formula: Amount of work (A) = rate (r) times time (t). Rewrite the equation as the rational equation isolating r: \( r = \frac{A}{t} \). Mary plants flowers at a rate of 200 seeds per hour. How many seeds has she planted in 2 hours? Write the rational equation and answer in a sentence.

\( 200 = \frac{A}{2} \)

Mary planted 400 seeds in 2 hours.

6. If one whole job can be accomplished in \( t \) units of time, then the rate of work is \( r = \frac{1}{t} \). Harry and Melanie are working on Lake Pontchartrain clean-up detail.

(a) Harry can clean up the trash in his area in 6 hours. Write an equation for Harry’s rate. \( r = \frac{1}{6} \)

(b) Melanie can do the same job in 4 hours. Write an equation for Melanie’s rate. \( r = \frac{1}{4} \)

(c) If they work together, how long will it take them to clean that area? Write a rational equation for the job and solve. \( \frac{1}{6} + \frac{1}{4} = \frac{1}{t} \), \( t = 2.4 \), It will take them 2.4 hours to clean the area if they work together.
## End-Behavior Around Vertical Asymptotes

This discovery worksheet will explore how the factors in the denominator and their exponents affect the graphs of equations in the form \( y = \frac{1}{(x - a)^n} \). All the graphs below have a horizontal asymptote at \( y = 0 \). Suggested calculator window: \( x: [-5, 5], y: [-5, 5] \)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Sketch</th>
<th>Vertical Asymptotes</th>
<th>( y )-intercept</th>
</tr>
</thead>
</table>
| **Example:**  
\( y = \frac{1}{x-2} \) | ![Sketch](example Sketch.png) | \( x = 2 \) | \( (0, -\frac{1}{2}) \) |
| (1)  
\( y = \frac{1}{(x-2)^2} \) | ![Sketch](example Sketch.png) | | |
| (2)  
\( y = \frac{1}{(x-2)^3} \) | ![Sketch](example Sketch.png) | | |
| (3)  
\( y = \frac{1}{(x-2)^4} \) | ![Sketch](example Sketch.png) | | |
| (4)  
\( y = \frac{1}{(x-2)(x+3)} \) | ![Sketch](example Sketch.png) | | (Hint: Zoom box around \( x = -3 \) to check graph) |
Unit 3, Activity 7, Vertical Asymptotes Discovery Worksheet

Use the results from the graphs above to answer the following questions:

1) How do you determine where the vertical asymptote is located?

2) What are the equations for the following vertical asymptotes?

(a) \( y = \frac{1}{(x-4)(x+3)(x-2)^2} \)  
(b) \( y = \frac{1}{2x^2 + 7x - 15} \)

3) What effect does the degree on the factor in the denominator have on the graph?

4) Predict the graphs of the following equation and then check on your calculator:

(a) \( y = \frac{40}{(x-5)^2(x-8)^3} \)  
(b) \( y = \frac{1}{x+2} \)  
(c) \( y = \frac{-1}{x+2} \)

(d) \( y = \frac{1}{x^2 - 9} \)  
(e) \( y = \frac{1}{(x-3)^2} \)  
(f) \( y = \frac{x-2}{x^2 - 8x + 12} \)
## End-Behavior Around Vertical Asymptotes

This discovery worksheet will explore how the factors in the denominator and their exponents effect the graphs of equations in the form \( y = \frac{1}{(x-a)^n} \). All the graphs below have a horizontal asymptote at \( y = 0 \). Suggested range (window): \( x: [-5,5], y: [-5,5] \)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Sketch</th>
<th>Vertical Asymptotes</th>
<th>y intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example:</td>
<td></td>
<td>( x = 2 )</td>
<td>( (0, -\frac{1}{2}) )</td>
</tr>
<tr>
<td>( y = \frac{1}{x-2} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) ( y = \frac{1}{(x-2)^2} )</td>
<td></td>
<td>( x = 2 )</td>
<td>( (0, \frac{1}{4}) )</td>
</tr>
<tr>
<td>(2) ( y = \frac{1}{(x-2)^3} )</td>
<td></td>
<td>( x = 2 )</td>
<td>( (0, -\frac{1}{8}) )</td>
</tr>
<tr>
<td>(3) ( y = \frac{1}{(x-2)^4} )</td>
<td></td>
<td>( x = 2 )</td>
<td>( (0, \frac{1}{16}) )</td>
</tr>
<tr>
<td>(4) ( y = \frac{1}{(x-2)(x+3)} )</td>
<td>(Hint: Zoom box around ( x = -3 ) to check graph)</td>
<td>( x = 2 ) ( x = -3 )</td>
<td>( (0, -\frac{1}{6}) )</td>
</tr>
</tbody>
</table>
Unit 3, Activity 7, Vertical Asymptotes Discovery Worksheet with Answers

Use the results from the graphs above to answer the following questions:

1) How do you determine where the vertical asymptote is located? Set the denominator = 0 and solve.

2) What are the equations for the following vertical asymptotes?

(a) \[ y = \frac{1}{(x-4)(x+3)(x-2)^2} \quad x = 4, x = -3, x = 2 \]  
(b) \[ y = \frac{1}{2x^2 + 7x - 15} \quad x = \frac{3}{2}, x = -5 \]

3) What effect does the degree on the factor in the denominator have on the graph?

*If the degree on the factor in the denominator is even, then the y-values approach the same infinity on either side of the asymptote formed by that factor. If the degree of the factor in the denominator is odd, the y-values approach opposite infinities on either side of the asymptote formed by that factor.*

4) Predict the graphs of the following equation and then check on your calculator:

(a) \[ y = \frac{40}{(x-5)^2(x-8)^3} \]  
(b) \[ y = \frac{1}{x+2} \]  
(c) \[ y = \frac{-1}{x+2} \]

(d) \[ y = \frac{1}{x^2 - 9} \]  
(e) \[ y = \frac{1}{(x-3)^2} \]  
(f) \[ y = \frac{x-2}{x^2 - 8x + 12} \]
Light at a Distance: Distance and Light Intensity

While traveling in a car at night, you may have observed the headlights of an oncoming vehicle. The light starts as a dim glow in the distance, but as the vehicle gets closer, the brightness of the headlights increases rapidly. This is because the light spreads out as it moves away from the source. As a result, light intensity decreases as the distance from a typical light source increases. What is the relationship between distance and intensity for a simple light bulb?

In this activity you can explore the relationship between distance and intensity for a light bulb. You will record the intensity at various distances between a Light Sensor and the bulb. The data can then be analyzed and modeled mathematically.

OBJECTIVES

- Collect light Intensity versus distance data for a point light source.
- Compare data to an inverse–square model
- Compare data to a power law model
- Discuss the difference between an inverse–square model and a power law model

MATERIALS

- TI-83 Plus or TI-84 Plus graphing calculator
- EasyData application
- data-collection interface
- Light Sensor
- meter stick or tape measure
- dc-powered point light source

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PROCEDURE

1. Arrange the equipment. There must be no obstructions between the bulb and the Light Sensor during data collection. Remove any surfaces near the bulb, such as books, people, walls or tables. There should be no reflective surfaces behind, beside, or below the bulb. The filament and Light Sensor should be at the same vertical height. This makes the light bulb look more like a point source of light as seen by the Light Sensor. While you are taking intensity readings, the Light Sensor must be pointed directly at the light bulb.

2. Set up the Light Sensor for data collection.
   a. Turn on the calculator.
   b. If you are using the Vernier Light Sensor, set it to 0–600 lux for a small light source, or 0–6000 lux for a larger light source.
   c. Connect the Light Sensor, data-collection interface, and calculator.

3. Set up EasyData for data collection.
   1. Start the EasyData application, if it is not already running.
   2. Select File from the Main screen, and then select New to reset the application.
   3. Select Setup from the Main screen, and then select Events with Entry.

4. Dim the lights to darken the room. A very dark room is critical to obtain good results.

5. Hold the Light Sensor about 10 cm from the light bulb filament. Move the sensor away from the bulb and watch the displayed intensity values on the calculator screen.

⇒ Answer Question 1 on the Data Collection and Analysis sheet.

6. To account for the particular brightness of your light source, choose a starting distance that gives a reading less than the maximum reading for your sensor (600 or 6000 lux for the Vernier sensor, or 1 for the TI sensor), but as large as possible. However, do not get any closer than 5 cm for small (<5 mm) bulbs, or 10 cm otherwise. Choose the starting distance, and enter it as $X_L$ in the Data Table on the Data Collection and Analysis sheet.

7. Again place the Light Sensor at your planned starting distance from the light bulb filament. Important: The distance must be measured carefully. Be sure you measure from the filament of the lamp to the sensor tip on the Light Sensor.

8. Select Start from the Main screen to prepare for data collection.

9. Wait for the value displayed on the calculator to stabilize. Select Keep and then enter the distance between the Light Sensor and the light source in meters on the calculator. Select OK to store the data pair.

10. Move the Light Sensor 1 cm farther away from the light source and repeat Step 9.

11. Continue moving the sensor in 1-cm increments until the readings fall to less than 10% of the Stop initial reading, collecting data as before. After the final data point, select to end data collection.
12. Inspect the graph of light intensity versus distance. Trace to read the \( x \) and \( y \) values of the left-most point, round the values to three significant figures, and record them as \( X_L \) and \( Y_L \) in the Data Table.

13. Select **Main** to return to the Main screen. Exit **EasyData** by selecting **Quit** from the Main screen and then selecting **OK**.

**ANALYSIS**

1. Redisplay your graph outside of **EasyData**.
   a. Press \( 2^{\text{ND}} \) [STAT PLOT] (above **Y=**).
   b. Press **ENTER** to select Plot1 and then press **ENTER** again to select On.
   c. Press **ZOOM**.
   d. Press \( \downarrow \) until ZoomStat is highlighted; press **ENTER** to display a graph with the \( x \) and \( y \) ranges set to fill the screen with data.

2. Inspect your graph of the light intensity versus distance.
   \( \Rightarrow \) Answer Question 2 on the Data Collection and Analysis sheet.

3. One model for light intensity holds that the intensity is proportional to the inverse square of the distance from a point light source; that is, a graph would be of the form \( y = \frac{C}{x^2} \), where \( C \) is an adjustable parameter. Does your data follow this model? You can check it out by finding an approximate value for \( C \), and then graphing the model with the data. First, enter the model equation.
   a. Press **Y=**
   b. Press **CLEAR** to remove any existing equation.
   c. Enter \( \frac{C}{x^2} \) in the \( Y_1 \) field.
   d. Press \( 2^{\text{ND}} \) [QUIT] (above **MODE**) to return to the home screen.

4. Set a value for the parameter \( C \) and then look at the resulting graph. To obtain a good fit, you will need to try several values for \( C \). Use the steps below to determine an initial guess for the parameter. One way to find an approximate value for the parameter is to use the left-most point. If you solve for \( C \), then \( C = yy^2 \). Use the \( x \) and \( y \) values for the left-most point to calculate an initial value for \( C \). Record this value in the Data Table on the Data Collection and Analysis sheet.
   a. Enter a value for the parameter \( C \). Press **STO \rightarrow C ENTER** to store the value in the variable \( C \).
   b. Press **GRAPH** to see the data with the model graph superimposed.
   c. Press \( 2^{\text{ND}} \) [QUIT] (above **MODE**) to return to the home screen.

5. If the model is systematically high or low, you may want to adjust the value of \( C \) to improve the fit. As you did before, store a new value in \( C \), and then display the graph and model. Once you have optimized the model, record the complete equation in the Data Table.
   \( \Rightarrow \) Answer Question 3 on the Data Collection and Analysis sheet.
6. Another model can be used to compare to your data. The general power law of $y = ax^b$ may provide a better fit than the inverse square function, especially if the light source is not small or if there are reflections from walls or other surfaces. The difference between this new model and the inverse square model is that the exponent is not fixed at $-2$. Instead, the exponent is now an adjustable parameter. The calculator can be used to automatically determine the parameters $a$ and $b$ in the general power law relation to the data.

a. Press STAT and use the cursor keys to highlight CALC.

b. Press ▼ repeatedly to scroll down to PwrReg. When it is highlighted, press ENTER to copy the command to the home screen.

c. Press 2ND [L1] 2ND [L2] to enter the lists containing the data.

d. Press VARS and use the cursor keys to highlight Y-VARS.

e. Select Function by pressing ENTER.

f. Press ENTER to copy Y1 to the home screen.

g. On the home screen, you will now see the entry PwrReg L1, L2, Y1. This command will perform a power law regression with L1 as the x and L2 as the y values. The resulting regression curve will be stored in equation variable Y1. Press GRAPH to perform the regression. Use the parameters a and b, rounded to two significant figures, to write the power law model equation in the Data Table.

h. Press GRAPH to see the graph of your data and the power regression function.

⇒ Answer Questions 4 and 5 on the Data Collection and Analysis sheet.

EXTENSIONS

1. Suppose that your patio is illuminated by an overhead light fixture with two bulbs. You decide to save on electricity by removing one of the bulbs. If the light is currently mounted 5 m off the ground, to what height should the light fixture be moved in order to retain the same amount of light on the patio with one bulb? Does your answer depend on the model you use?

2. Two identical light bulbs shine on your favorite reading chair from different locations in the room. The first bulb is 3 m from your chair and provides an intensity of 0.6 mW/cm². The second is 2 m from your chair. What intensity does this bulb provide? Does your answer depend on the model you use?
DATA COLLECTION AND ANALYSIS

Name ____________________________
Date ____________________________

DATA TABLES

<table>
<thead>
<tr>
<th>Left-most point X value X_l</th>
<th>Left-most point Y value Y_l</th>
<th>Initial model parameter C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Optimized inverse-square model \( y = \frac{C}{x^2} \)

Power law model \( y = ax^b \)

QUESTIONS

1. What is your prediction for the relationship between intensity and the distance to a light source?

2. Is the graph of the light intensity versus distance consistent with your earlier prediction?

3. How well does the inverse-square model \( y = \frac{C}{x^2} \) fit the experimental data?

4. How well does the power law model fit the data? Could it fit any better than the inverse-square model? Could it fit more poorly?

5. How would using a brighter light bulb affect the parameters \( a, b, \) and \( C \) in the two models?
Light at a Distance:  
Distance and Light Intensity

1. There are currently 2 different combinations of equipment that will work for collecting light data. The most common method, which works for both the TI-83 Plus and TI-84 Plus families of calculators, is to use a Light Sensor attached to a CBL 2 or LabPro.

   The TI-84 Plus calculator has a USB port located at the top right corner. Using the USB port, an EasyLink with a Light Sensor can be connected to collect light data. For more information on EasyLink refer to Page ix located in the front section of this book.

2. When connecting an EasyLink to a TI-84 Plus calculator using USB, the EasyData application automatically launches when the calculator is turned on and at the home screen.

3. If you obtain readings of more than 600 or 6000 lux (Vernier sensor, depending on setting) or 1 (TI sensor) move farther away from the light source. Students may need to adjust the range of distances used for data collection, depending on the brightness of the light source. Some students may need help with this in Step 8 of the Procedure. Because of the different light sources that could be used in the lab, the optimum range for data collection will vary.

4. Only a true point light source exhibits an inverse-square dependence of intensity on distance. It is very difficult in a classroom to achieve a true point light source with no reflective surfaces nearby. As a result, you and your student should not consider results incorrect if you do not get an exponent of nearly −2 in the power law fit. In fact, it is unlikely you will obtain a −2 exponent. An extended light source will yield an exponent between −1 and −2. A long straight light, such as a fluorescent tube, will yield an exponent of about −1 at typical distance ranges.

5. An excellent light source for this experiment is the AA-cell size Mini Mag Lite (www.maglite.com). The reflector can be unscrewed and removed completely, revealing a very small and intense near-point light. If another kind of flashlight is used, it is essential that it not have a reflector around the bulb, or the source will not behave at all like a point source.

6. It is important that the light be powered by DC (direct current), such as by a battery source. AC-powered lamps exhibit a time-varying flicker that is not detectable by eye, but that may substantially reduce the quality of the data.

7. The quantity measured by the light sensors is not strictly called “intensity” but we use this common term for convenience.

8. You may want to have students adjust the window range to replot the data to include the origin. The inverse-square nature of the model is more readily visible when plotted this way. (See sample data below.)
Activity 16

SAMPLE RESULTS

![Sample data with inverse-square model](image1)

![Same sample data replotted over a wider range](image2)

![Sample data with power law model](image3)

![Power law fit parameters](image4)

DATA TABLE

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>Left-most point Y value Y&lt;sub&gt;L&lt;/sub&gt;</td>
<td>244</td>
</tr>
<tr>
<td>Initial model parameter C</td>
<td>6100</td>
</tr>
<tr>
<td>Optimized inverse-square model y = C/x&lt;sup&gt;2&lt;/sup&gt;</td>
<td>y = 6300/x&lt;sup&gt;2&lt;/sup&gt;</td>
</tr>
<tr>
<td>Power law model y = ax&lt;sup&gt;b&lt;/sup&gt;</td>
<td>y = 4100x&lt;sup&gt;-1.8&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

ANSWERS TO QUESTIONS

1. The intensity of the light appears to get smaller with increasing distance. The relationship could be an inverse relationship, or it could be something else.

2. Yes, the data are consistent with a decreasing function such as an inverse function, as the graph is always decreasing but it never crosses the horizontal axis.

3. The inverse-square model fits the data very well.

4. The fit of the general power law looks very similar to the inverse-square fit. The power law fit has to be at least as good as the inverse-square fit, since the power law fit includes the possibility of the inverse-square fit with the exponent taking a value of –2. The fit of the power law model could be better than the inverse-square model if the data don’t actually have an inverse-square behavior.

5. A brighter bulb would shift increase all readings proportionately. So, a and C would increase, but b would stay the same.
**Unit 3, Activity 9, Rational Inequalities**

Name_________________________________________ Date________________________

**Using a Sign Chart to Solve Rational Inequalities**

Solve the following using a sign chart

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) (\frac{(x-3)(x-4)}{(x-5)(x-6)^2} \leq 0)</td>
<td>(6) (\frac{5}{x+2} \geq 1)</td>
</tr>
<tr>
<td>(2) (\frac{x+4}{x-2} \geq 0)</td>
<td>(7) (\frac{1}{x-4} \geq \frac{x}{x-6})</td>
</tr>
<tr>
<td>(3) (\frac{x^2+1}{x-1} &lt; 0)</td>
<td>(8) (\frac{1}{4a} + \frac{5}{8a} &gt; \frac{1}{2})</td>
</tr>
<tr>
<td>(4) (\frac{x^2-16}{x^2-4x-5} &gt; 0)</td>
<td>(9) (\frac{2}{x} &lt; x + 1)</td>
</tr>
<tr>
<td>(5) (\frac{x^2-x-6}{(x-1)(x+1)^2} &gt; 0)</td>
<td>(10) (\frac{1}{3b} - \frac{2}{5b} \geq 1)</td>
</tr>
</tbody>
</table>

**Application**

The production of heating oil produced by an oil refinery depends on the amount of gasoline produced. The amount of heating oil produced (in gallons per day) is modeled by the rational function

\[ h(g) = \frac{125,000 - 25g}{125 + 2g} \]

where \(g\) is the amount of gasoline produced (in hundreds of gallons per day). If customers need more than 300 gallons of heating oil per day, how many gallons of gasoline will be produced?
Unit 3, Activity 9, Rational Inequalities with Answers

Using a Sign Chart to Solve Rational Inequalities
Solve the following using a sign chart

1) \( \frac{(x - 3)(x - 4)}{(x - 5)(x - 6)} \leq 0 \)
   \((-\infty, 3] \cup [4, 5)\)

2) \( \frac{x + 4}{x - 2} \geq 0 \)
   \( (\infty, -4] \cup (2, \infty) \)

3) \( \frac{x^2 + 1}{x - 1} < 0 \)
   \((-\infty, 1)\)

4) \( \frac{x^2 - 16}{x^2 - 4x - 5} > 0 \)
   \((-\infty, -4) \cup (-1, 4) \cup (5, \infty)\)

5) \( \frac{x^2 - x - 6}{(x - 1)(x + 1)} \geq 0 \)
   \((-2, -1) \cup (-1, 1) \cup (3, \infty)\)

6) \( \frac{5}{x + 2} \geq 1 \)
   \((-2, \infty)\)

7) \( \frac{1}{x - 4} \geq \frac{x}{x - 6} \)
   \( [2, 3] \cup (4, 6) \)

8) \( \frac{1}{4a} + \frac{5}{8a} > \frac{1}{2} \)
   \( \left(0, \frac{7}{4}\right) \)

9) \( \frac{2}{x} < x + 1 \)
   \((-1, 0) \cup (2, \infty)\)

10) \( \frac{1}{5b} - \frac{2}{5b} \geq 1 \)
    \( \left(-\infty, \frac{11}{15}\right) \cup (0, \infty)\)

Application

The production of heating oil produced by an oil refinery depends on the amount of gasoline produced. The amount of heating oil produced (in gallons per day) is modeled by the rational function \( h(g) = \frac{125,000 - 25g}{125 + 2g} \) where g is the amount of gasoline produced (in hundreds of gallons per day). If customers need more than 300 gallons of heating oil per day, how many gallons of gasoline will be produced?

\( \frac{125,000 - 25g}{125 + 2g} > 300 \) Less than 199.4 gallons of gasoline will be produced.
4.1 Radical Terminology – define radical sign, radicand, index, like radicals, root, n\textsuperscript{th} root, principal root, conjugate.

4.2 Rules for Simplifying $\sqrt[n]{b^n}$ – identify and give examples of the rules for even and odd values of $n$.

4.3 Product and Quotient Rules for Radicals – identify and give examples of the rules.

4.4 Rationalizing the Denominator – explain: what does it mean, why do it, the process for rationalizing a denominator of radicals with varying indices and a denominator that contains the sum of two radicals.

4.5 Radicals in Simplest Form – list what to check for to make sure radicals are in simplest form.

4.6 Addition and Subtraction Rules for Radicals – identify and give examples.

4.7 Graphing Simple Radical Functions – show the effect of constant both inside and outside of a radical on the domain and range.

4.8 Steps to Solve Radical Equations – identify and give examples.

4.9 Complex Numbers – define: $i$, $a + bi$ form, $i$, $i^2$, $i^3$, and $i^4$; explain how to find the value of $i^{4n}$, $i^{4n+1}$, $i^{4n+2}$, $i^{4n+3}$, explain how to conjugate and find the absolute value of $a + bi$.

4.10 Properties of Complex Number System – provide examples of the equality property, the commutative property under addition/multiplication, the associative property under addition/multiplication, and the closure property under addition/multiplication.

4.11 Operations on Complex Numbers in $a + bi$ form – provide examples of addition, additive identity, additive inverse, subtraction, multiplication, multiplicative identity, squaring, division, absolute value, reciprocal, raising to a power, and factoring the sum of two perfect squares.

4.12 Root vs. Zero – explain the difference between a root and a zero and how to determine the number of roots of a polynomial.
Graph on the graphing calculator and find the points of intersection:

(1) \( y_1 = x^2 \) and \( y_2 = 9 \)
(2) \( y_1 = x^2 \) and \( y_2 = -9 \)
(3) \( y_1 = x^2 \) and \( y_2 = 0 \)
(4) Discuss the number of points of intersection each set of equations has.
Unit 4, Activity 2, Sets of Numbers

Reviewing Sets of Numbers

Fill in the following sets of numbers in the Venn diagram: natural numbers, whole numbers, integers, rational numbers, irrational numbers, real numbers.

1. Write the symbol for the set and list its elements in set notation:
   - natural numbers: _______________ What is another name for natural numbers? _______
   - whole numbers: __________________________
   - integers: __________________________

2. Define rational numbers. What is its symbol and why? Give some examples. __________

3. Are your Bellringers rational or irrational? Why? __________________________
Reviewing Sets of Numbers

Fill in the following sets of numbers in the Venn diagram: natural numbers, whole numbers, integers, rational numbers, irrational numbers, real numbers.

1. Write the symbol for the set and list its elements in set notation:
   - natural numbers: \( N = \{1, 2, 3, \ldots\} \) What is another name for natural numbers? **Counting**
   - whole numbers: \( W = \{0, 1, 2, 3, \ldots\} \)
   - integers: \( \mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} \)

2. Define rational numbers. What is its symbol and why? Give some examples. **Any number in the form \( \frac{p}{q} \) where \( p \) and \( q \) are integers, \( q \neq 0 \). The symbol is \( \mathbb{Q} \) for quotient. Ex. All repeating and terminating decimals and fractions of integers. 7, 7.5, 7.6666..., ½, −1/3**

3. Are your Bellringer problems rational or irrational? Why? **Irrational because they cannot be expressed as fractions of integers. Their decimal representations do not repeat or terminate.**
**Unit 4, Activity 2, Multiplying & Dividing Radicals**

Name_________________________ Date_________________________

**Multiplying and Dividing Radicals**

1. Can the product of two irrational numbers be a rational number? Give an example.  
   __________________________________________________________________________

2. What does “rationalizing the denominator” mean and why do we rationalize the denominator?  
   __________________________________________________________________________

3. Rationalize the following denominators and simplify:
   
   (1) \( \frac{1}{\sqrt{5}} \)  
   (2) \( \frac{1}{\sqrt[3]{5}} \)  
   (3) \( \frac{1}{\sqrt{8}} \)

4. List what should be checked to make sure a radical is in simplest form:
   
   a.  
   __________________________________________________________________________
   
   b.  
   __________________________________________________________________________
   
   c.  
   __________________________________________________________________________

5. Simplify the following expressions applying rules to radicals with variables in the radicand.
   
   (1) \( \sqrt[3]{72x^3y^4} \)  
   (2) \( \sqrt[3]{80s^4t^6} \)  
   (3) \( \sqrt{2xy} \cdot \sqrt{6x^3y} \)  
   (4) \( 5\sqrt[3]{4x^2y^5} \cdot 7\sqrt[3]{2x^2y} \)  
   (5) \( \frac{\sqrt{162x^6}}{\sqrt[3]{10x^7}} \)  
   (6) \( \frac{\sqrt[3]{2s^2}}{\sqrt[3]{18s^3}} \)

**Application**

The time in seconds, \( t(L) \), for one complete swing of a pendulum is dependent upon the length of the pendulum in feet, \( L \), and gravity which is 32 ft/sec\(^2\) on earth. It is modeled by the function \( t(L) = 2\pi \sqrt[3]{\frac{L}{32}} \). Find the time for one complete swing of a 4-foot pendulum.

Express the exact simplified answer in function notation and express the answer in a sentence rounding to the nearest tenth of a second.
Multiplying and Dividing Radicals

1. Can the product of two irrational numbers be a rational number? Give an example.

   Yes, \( \sqrt{2} \times \sqrt{8} = 8 \)

2. What does “rationalizing the denominator” mean and why do we rationalize the denominator?

   Rationalizing the denominator means making sure that the number in the denominator is a rational number and not an irrational number with a radical. We rationalize denominators because we do not want to divide by a nonrepeating, nonterminating decimal.

3. Rationalize the following denominators and simplify:
   
   (1) \( \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} \)
   
   (2) \( \frac{1}{\sqrt{6}} = \frac{\sqrt{25}}{5} \)
   
   (3) \( \frac{1}{\sqrt{8}} = \frac{\sqrt{2}}{4} \)

4. List what should be checked to make sure a radical is in simplest form:

   a. The radicand contains no exponent greater than or equal to the index
   
   b. The radicand contains no fractions
   
   c. The denominator contains no radicals

5. Simplify the following expressions applying rules to radicals with variables in the radicand.

   (1) \( \sqrt{72x^3y^4} = 6y^2|x|\sqrt{2x} \)
   
   (2) \( \sqrt[3]{80s^4t^6} = 2st^2\sqrt[3]{10s} \)
   
   (3) \( \sqrt{2xy} \cdot \sqrt{6x^3y} = 2x^2|y|\sqrt{y} \)
   
   (4) \( 5\sqrt{4x^2y^5} \cdot 7\sqrt{2x^2y} = 70xy^2\sqrt{x} \)
   
   (5) \( \frac{\sqrt{162x^6}}{\sqrt{10x^7}} = \frac{9\sqrt{x}}{5x} \)
   
   (6) \( \frac{\sqrt{2x^7}}{\sqrt{18s^3}} = \frac{\sqrt{3s^2}}{3s} \)

Application

The time in seconds, \( t(L) \), for one complete swing of a pendulum is dependent upon the length of the pendulum in feet, \( L \), and gravity which is 32 ft/sec\(^2\) on earth. It is modeled by the function \( t(L) = 2\pi \sqrt{\frac{L}{32}} \). Find the time for one complete swing of a 4-foot pendulum.

Express the exact simplified answer in function notation and express the answer in a sentence rounding to the nearest tenth of a second. \( t(4) = \frac{\pi \sqrt{2}}{2} \). One complete swing of a 4-foot pendulum takes approximately 2.2 seconds.
**Unit 4, Activity 4, Graphing Radical Functions Discovery Worksheet**

**Radical Graph Translations**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Sketch</th>
<th>Domain</th>
<th>Range</th>
<th>x-intercept</th>
<th>y-intercept</th>
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</tr>
<tr>
<td>$y = \sqrt{x} - 3 + 5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

(10) What is the difference in the graph when a constant is added outside of the radical, $f(x) + k$, or inside of the radical, $f(x + k)$?

(11) What is the difference in the domains and ranges of $f(x) = \sqrt{x}$ and $g(x) = \sqrt[3]{x}$? Why is the domain of one of the functions restricted and the other not?
### Radical Graph Translations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Sketch</th>
<th>Domain</th>
<th>Range</th>
<th>x-intercept</th>
<th>y-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( y = \sqrt{x} + 3 )</td>
<td></td>
<td>( x \geq 0 )</td>
<td>( y \geq 3 )</td>
<td>none</td>
<td>(0, 3)</td>
</tr>
<tr>
<td>2 ( y = \sqrt{x} - 3 )</td>
<td></td>
<td>( x \geq 0 )</td>
<td>( y \geq -3 )</td>
<td>(9, 0)</td>
<td>(0, -3)</td>
</tr>
<tr>
<td>3 ( y = \frac{1}{2}\sqrt{x} + 2 )</td>
<td></td>
<td>all reals</td>
<td>all reals</td>
<td>(-8, 0)</td>
<td>(0, 2)</td>
</tr>
<tr>
<td>4 ( y = \frac{1}{3}\sqrt{x} - 2 )</td>
<td></td>
<td>all reals</td>
<td>all reals</td>
<td>(8, 0)</td>
<td>(0, -2)</td>
</tr>
<tr>
<td>5 ( y = \sqrt{x} - 4 )</td>
<td></td>
<td>( x \geq 4 )</td>
<td>( y \geq 0 )</td>
<td>(4, 0)</td>
<td>none</td>
</tr>
<tr>
<td>6 ( y = \sqrt{x} + 4 )</td>
<td></td>
<td>( x \geq -4 )</td>
<td>( y \geq 0 )</td>
<td>(-4, 0)</td>
<td>(0, 2)</td>
</tr>
<tr>
<td>7 ( y = \frac{1}{2}\sqrt{x} + 5 )</td>
<td></td>
<td>all reals</td>
<td>all reals</td>
<td>(-5, 0)</td>
<td>(0, ( \sqrt{5} ))</td>
</tr>
<tr>
<td>8 ( y = \frac{1}{3}\sqrt{x} - 5 )</td>
<td></td>
<td>all reals</td>
<td>all reals</td>
<td>(5, 0)</td>
<td>(0, ( \sqrt[3]{5} ))</td>
</tr>
<tr>
<td>9 ( y = \sqrt{x} - 3 + 5 )</td>
<td></td>
<td>( x \geq 3 )</td>
<td>( y \geq 5 )</td>
<td>none</td>
<td>none</td>
</tr>
</tbody>
</table>

10. What is the difference in the graph when a constant is added outside of the radical, \( f(x) + k \), or inside of the radical, \( f(x + k) \)? Outside the radical changes the vertical shift, + up and – down. A constant inside the radical, changes the horizontal shift, + left and n right.

11. What is the difference in the domains and ranges of \( f(x) = \sqrt{x} \) and \( g(x) = \sqrt[3]{x} \)? Why is the domain of one of the functions restricted and the other not? Even index radicals have a restricted domain \( x \geq 0 \) and therefore a resulting restricted range \( y \geq 0 \). The domain and range of odd index radicals are both all reals. You cannot take an even index radical of a negative number.
**Complex Number System Word Grid**

Place an “X” in the box corresponding to the set to which the number belongs:

<table>
<thead>
<tr>
<th>Natural #</th>
<th>Whole #</th>
<th>Integer</th>
<th>Rational #</th>
<th>Irrational #</th>
<th>Real #</th>
<th>Imaginary #</th>
<th>Complex #</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>−5</td>
<td>3</td>
<td>0</td>
<td>28</td>
<td>5.7</td>
<td>4.16</td>
<td>π</td>
</tr>
<tr>
<td>½</td>
<td>√5</td>
<td>3√5</td>
<td>6 + 3√5</td>
<td>i</td>
<td>√−4</td>
<td>6i√2</td>
<td>2 + 3i</td>
</tr>
</tbody>
</table>

**Properties of the Complex Number System**

When creating any new number system, certain mathematical properties and operations must be defined. Your team will be assigned some of the following properties. On the transparency or chart paper, define the property for the Complex Number System in words (verbally) and using $a + bi$ (symbolically) and give a complex number example without using the book. Each member of your team will present one of the properties to the class, and the class will decide if it is correct. The team with the most Best Properties wins a bonus point (or candy, etc.).

Sample:

<table>
<thead>
<tr>
<th>Properties/Operations</th>
<th>Defined verbally and symbolically with a complex number example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equality of Complex Numbers</td>
<td>Two complex numbers are equal if the real parts are equal and the imaginary parts are equal.</td>
</tr>
<tr>
<td></td>
<td>$a + bi = c + di$ if and only if $a = c$ and $b = d$.</td>
</tr>
<tr>
<td></td>
<td>$6 + 3i = \sqrt{36} + \sqrt{-9}$</td>
</tr>
</tbody>
</table>

**Properties and operations:** addition, additive identity, additive inverse, subtraction, multiplication, multiplicative identity, squaring, dividing, absolute value, reciprocal (multiplicative inverse), commutative under addition and multiplication, associative under addition/multiplication, closed under addition and multiplication, factoring the difference in two perfect squares, factoring the sum of two perfect squares.
Unit 4, Activity 7, Complex Number System with Answers

Complex Number System Word Grid

Place an “X” in the box corresponding to the set to which the number belongs:

<table>
<thead>
<tr>
<th></th>
<th>10</th>
<th>-5</th>
<th>3</th>
<th>0</th>
<th>28</th>
<th>5.7</th>
<th>π</th>
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<th>2+3i</th>
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</thead>
<tbody>
<tr>
<td>Natural #</td>
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</tr>
</tbody>
</table>

Optional sets of numbers for discussion:
- algebraic number ≡ real # that occurs as root of a polynomial equation that have integer coefficients.
- transcendental number ≡ not algebraic
- perfect number ≡ any natural number which is equal to the sum of its divisors < itself such as 6 = 1 + 2 + 3
- prime number ≡ any number that can be divided, without a remainder, only by itself and 1
- composite number ≡ any number that is a multiple of two numbers other than itself and 1
- surds ≡ an irrational number that can be expressed as a radical

Properties of the Complex Number System

When creating any new number system, certain mathematical properties and operations must be defined. Your team will be assigned some of the following properties. On the transparency or chart paper, define the property for the Complex Number System in words (verbally) and using $a + bi$ (symbolically) and give a complex number example without using the book. Each member of your team will present one of the properties to the class, and the class will decide if it is correct. The team with the most Best Properties wins a bonus point (or candy, etc.).

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Do You Really Know the Difference?

State whether the following numbers are real (R) or imaginary (I) and discuss why.

(1) \(i\)  
(2) \(i^2\)  
(3) \(\sqrt{-9}\)  
(4) \(\sqrt{-2} \sqrt{-5}\)  
(5) \(i^n\) if \(n\) is even  
(6) the sum of an imaginary number and its conjugate  
(7) the difference of an imaginary number and its conjugate  
(8) the product of an imaginary number and its conjugate  
(9) the conjugate of an imaginary number  
(10) the conjugate of a real number  
(11) the reciprocal of an imaginary number  
(12) the additive inverse of an imaginary number  
(13) the multiplicative identity of an imaginary number  
(14) the additive identity of an imaginary number

Answers:

(1) __  
(2) __  
(3) __  
(4) __  
(5) __  
(6) __  
(7) __  
(8) __  
(9) __  
(10) __  
(11) __  
(12) __  
(13) __  
(14) __  

Blackline Masters, Algebra II
Louisiana Comprehensive Curriculum, Revised 2008
State whether the following numbers are real (R) or imaginary (I) and discuss why.

1. $i$
2. $i^2$
3. $\sqrt{-9}$
4. $\sqrt{-2}\sqrt{-5}$
5. $i^n$ if $n$ is even
6. The sum of an imaginary number and its conjugate
7. The difference of an imaginary number and its conjugate
8. The product of an imaginary number and its conjugate
9. The conjugate of an imaginary number
10. The conjugate of a real number
11. The reciprocal of an imaginary number
12. The additive inverse of an imaginary number
13. The multiplicative identity of an imaginary number
14. The additive identity of an imaginary number

Answers:

1. $I$ This is the imaginary number equal to $\sqrt{-1}$.
2. $R$ $i^2 = -1$ which is real.
3. $I$ $\sqrt{-9} = 3i$ which is imaginary.
4. $R$ $\sqrt{-2}\sqrt{-5} = (i\sqrt{2})(i\sqrt{5}) = i^2\sqrt{10} = -\sqrt{10}$ which is real.
5. $R$ If $n$ is even then $i^n$ will either be 1 or $-1$ which are real.
6. $R$ $(a + bi) + (a - bi) = 2a$ which is real.
7. $I$ $(a + bi) - (a - bi) = 2bi$ which is imaginary.
8. $R$ $(a + bi)(a - bi) = a^2 + b^2$ which is real.
9. $I$ The conjugate of $(0 + bi)$ is $(0 - bi)$ which is imaginary.
10. $R$ The conjugate of $(0 + bi)$ is $(0 - bi)$ which is real.
11. $I$ The reciprocal if $i$ is $\frac{1}{i}$ which equals $-i$ when you rationalize the denominator $-$ imaginary.
12. $I$ The additive inverse of $(0 + bi)$ is $(0 - bi)$ which is imaginary.
13. $R$ The multiplicative identity of $(0 + bi)$ is $(1 + 0i)$ which is real.
14. $R$ The additive identity of $(0 + bi)$ is $(0 + 0i)$ which is real.
5.1 **Quadratic Function** – give examples in standard form and demonstrate how to find the vertex and axis of symmetry.

5.2 **Translations and Shifts of Quadratic Functions** – discuss the effects of the symbol ± before the leading coefficient, the effect of the magnitude of the leading coefficient, the vertical shift of equation $y = x^2 \pm c$, the horizontal shift of $y = (x - c)^2$.

5.3 Three ways to Solve a Quadratic Equation – write one quadratic equation and show how to solve it by factoring, completing the square, and using the quadratic formula.

5.4 **Discriminant** – give the definition and indicate how it is used to determine the nature of the roots and the information that it provides about the graph of a quadratic equation.

5.5 **Factors, x-intercept, y-intercept, Roots, Zeros** – write definitions and explain the difference between a root and a zero.

5.6 **Comparing Linear functions to Quadratic Functions** – give examples to compare and contrast $y = mx + b$, $y = x(mx + b)$, and $y = x^2 + mx + b$, explain how to determine if data generates a linear or quadratic graph.

5.7 **How Varying the Coefficients in $y = ax^2 + bx + c$ Affects the Graph** – discuss and give examples.

5.8 **Quadratic Form** – define, explain, and give several examples.

5.9 **Solving Quadratic Inequalities** – show an example using a graph and a sign chart.

5.10 **Polynomial Function** – define polynomial function, degree of a polynomial, leading coefficient, and descending order.

5.11 **Synthetic Division** – identify the steps for using synthetic division to divide a polynomial by a binomial.

5.12 **Remainder Theorem, Factor Theorem** – state each theorem and give an explanation and example of each, explain how and why each is used, state their relationships to synthetic division and depressed equations.

5.13 **Fundamental Theorem of Algebra, Number of Roots Theorem** – give an example of each theorem.

5.14 **Intermediate Value Theorem** – state theorem and explain with a picture.

5.15 **Rational Root Theorem** – state the theorem and give an example.

5.16 **General Observations of Graphing a Polynomial** – explain the effects of even/odd degrees on graphs, explain the effect of the use of ± leading coefficient on even and odd degree polynomials, identify the number of zeros, explain and show an example of double root.

5.17 **Steps for Solving a Polynomial of 4th degree** – work all parts of a problem to find all roots and graph.
One side, $s$, of a rectangle is four inches less than the other side. Draw a rectangle with these sides and find an equation for the area $A(s)$ of the rectangle.
Unit 5, Activity 1, Zeroes of a Quadratic Function

Graph the function from the Bellringer \( y = x^2 - 4x \) on your calculators. This graph is called a parabola. Sketch the graph making sure to accurately find the \( x \) and \( y \) intercepts and the minimum value of the function.

(1) In the context of the Bellringer, what do the \( x \)-values represent? ___________  
the \( y \)-values? ___________

(2) From the graph, list the zeroes of the equation. ________________

(3) What is the real-world meaning of the zeroes for the Bellringer? ________________

(4) Solve for the zeroes analytically showing your work. What property of equations did you use to find the zeroes?

Local and Global Characteristics of a Parabola

(1) In your own words, define axis of symmetry: ________________________________

(2) Write the equation of the axis of symmetry in the graph above. ________________

(3) In your own words, define vertex: ________________________________

(4) What are the coordinates of the vertex of this parabola? ______________________

(5) What is the domain of the graph above? _______________ range? _______________

(6) What domain has meaning for the Bellringer and why? ______________________

(7) What range has meaning for the Bellringer and why? ______________________
**Unit 5, Activity 1, Zeroes of a Quadratic Function**

**Reviewing 2nd Degree Polynomial Graphs**

Graph the following equations and answer the questions in your notebook.

1. \( y = x^2 \) and \( y = -x^2 \). How does the sign of the leading coefficient affect the graph of the parabola?

2. \( y = x^2, y = 4x^2, y = 0.5x^2 \). How does the magnitude of the leading coefficient affect the zeroes and the shape of the parabola as compared to \( y = x^2 \)?

3. \( y = (x - 3)(x + 4), y = (x - 1)(x + 6) \). Make conjectures about the zeroes.

4. \( y = 2(x - 5)(x + 4), y = -2(x - 5)(x + 4) \). Make conjectures about the zeroes and end-behavior.

**Application**

A tunnel in the shape of a parabola over a two-lane highway has the following features. It is 30 feet wide at the base and 23 feet high in the center.

1. Make a sketch of the tunnel on a coordinate plane with the ground as the \( x \)-axis and the left side of the base of the tunnel at \( (2, 0) \). Find two more ordered pairs and graph as a scatter plot in your calculator.

2. Enter the quadratic equation \( y = a(x - b)(x - c) \) in your calculator substituting your \( x \)-intercepts from your sketch into \( b \) and \( c \). Experiment with various numbers for “\( a \)” to find the parabola that best fits this data. Write your equation.

3. An 8-foot wide 12-foot high truck wants to go through the tunnel. Determine whether the truck will fit and the allowable location of the truck. Explain your answer.
Graph the function from the Bellringer \( y = x^2 - 4x \) on your calculators. This graph is called a parabola. Sketch the graph making sure to accurately find the \( x \) and \( y \) intercepts and the minimum value of the function.

1. In the context of the Bellringer, what do the \( x \)-values represent? the length of the sides
   the \( y \)-values? the area

2. From the graph, list the zeroes of the equation. 0 and 4

3. What is the real-world meaning of the zeroes for the Bellringer?
   The length of the side for which the area is zero.

4. Solve for the zeroes analytically showing your work. What property of equations did you use to find the zeroes?
   \( 0 = x^2 - 4x \Rightarrow 0 = x(x - 4) \Rightarrow x = 0 \) or \( x - 4 = 0 \) by the Zero Property of Equations \( \Rightarrow \{0, 4\} \)

**Local and Global Characteristics of a Parabola**

1. In your own words, define axis of symmetry: a line about which pairs of points on the parabola are equidistant

2. Write the equation of the axis of symmetry in the graph above. \( x = 2 \)

3. In your own words, define vertex: The point where the parabola intersects the axis of symmetry

4. What are the coordinates of the vertex of this parabola? \((2, -4)\)

5. What is the domain of the graph above? all real numbers range? \( y \geq -4 \)

6. What domain has meaning for the Bellringer and why? \( x > 4 \) because those sides create positive area

7. What range has meaning for the Bellringer and why? \( y > 0 \) because you want an area > 0
Unit 5, Activity 1, Zeroes of a Quadratic Function with Answers

Reviewing 2nd Degree Polynomial Graphs

Graph the following equations and answer the questions in your notebook.
(1) $y = x^2$ and $y = -x^2$. How does the sign of the leading coefficient affect the graph of the parabola?
   - Even exponent polynomial has similar end behavior. Positive leading coefficient starts up and ends up, negative leading coefficient starts down and ends down.
(2) $y = x^2, y = 4x^2, y = 0.5x^2$. How does the magnitude of the leading coefficient affect the zeroes and the shape of the parabola as compared to $y = x^2$?
   - It does not affect the zeroes. If constant is > 1, the graph is steeper than $y = x^2$, and if the constant is less than 1, the graph is wider than $y = x^2$.
(3) $y = (x - 3)(x + 4), y = (x - 1)(x + 6)$. Make conjectures about the zeroes. When the function is factored, the zeroes of the parabola are at the solutions to the factors set = 0.
(4) $y = 2(x - 5)(x + 4), y = -2(x - 5)(x + 4)$. Make conjectures about the zeroes and end-behavior.
   - Same zeroes opposite end-behaviors.

Application

A tunnel in the shape of a parabola over a two-lane highway has the following features. It is 30 feet wide at the base and 23 feet high in the center.
(1) Make a sketch of the tunnel on a coordinate plane with the ground as the x-axis and the left side of the base of the tunnel at (2, 0). Find two more ordered pairs and graph as a scatter plot in your calculator. (32, 0) and (17, 23)

(2) Enter the quadratic equation $y = a(x - b)(x - c)$ in your calculator substituting your x-intercepts from your sketch into b and c. Experiment with various numbers for “a” to find the parabola that best fits this data. Write your equation.

   $y = -0.1(x - 2)(x - 32)$

(3) An 8-foot wide 12-foot high truck wants to go through the tunnel. Determine whether the truck will fit and the allowable location of the truck. Explain your answer.

   The truck must travel 4.75 feet from the base of the tunnel. It is 8 feet wide and the center of the tunnel is 15 feet from the base so the truck can stay in its lane.
Give your opinion of what will happen to the graphs in the following situations based upon your prior knowledge of translations and transformations of graphs.

(1) Predict what will happen to the graphs of form \( y = x^2 + 5x + c \) for the following values of \( c \): \{8, 4, 0, –4, –8\}.

(2) Predict what will happen to the graphs of form \( y = x^2 + bx + 4 \) for the following values of \( b \): \{6, 3, 0, –3, –6\}.

(3) Predict what will happen to the graphs of form \( y = ax^2 + 5x + 4 \) for the following values of \( a \): \{-2, –1, \(-\frac{1}{2}\), 0, \(\frac{1}{2}\), 1, 2\}. 
Unit 5, Activity 7, The Changing Parabola Discovery Worksheet

Name ___________________________ Date ______________________

(1) Graph \( y = x^2 + 5x + 4 \) which is in the form \( y = ax^2 + bx + c \) (without a calculator). Determine the following global characteristics:

Vertex: _______________ \( x \)-intercept: ______, \( y \)-intercept: ______

Domain: _______________ Range: _______________

End–behavior: __________________________________________

(2) Graph \( y = x^2 + 5x + c \) on your calculator for the following values of \( c \): {8, 4, 0, –4, –8} and sketch. (WINDOW: \( x: [-10, 10], y: [-15, 15] \))

- What special case occurs at \( c = 0 \)? _______________
- Check your predictions on your anticipation guide. Were you correct? Explain why the patterns occur.

(3) Graph \( y = x^2 + bx + 4 \) on your calculator for the following values of \( b \): {6, 3, 0, –3, –6} and sketch.

- What special case occurs at \( b = 0 \)? _______________
- Check your predictions on your anticipation guide. Were you correct? Explain why the patterns occur.

(4) Graph \( y = ax^2 + 5x + 4 \) on your calculator for the following values of \( a \): {2, 1, 0.5, 0, –0.5, –1, –2} and sketch.

- What special case occurs at \( a = 0 \)? _______________
- Check your predictions on your anticipation guide. Were you correct? Explain why the patterns occur.
(1) Graph \( y = x^2 + 5x + 4 \) which is in the form \( y = ax^2 + bx + c \) (without a calculator). Determine the following global characteristics:

- **Vertex:** \( \left( -\frac{5}{2}, -\frac{9}{4} \right) \)
- **x-intercept:** \( \{-4, -1\} \)
- **y-intercept:** \( \{4\} \)

**Domain:** **All Reals**  
**Range:** \( y \geq -\frac{9}{4} \)

End-behavior: as \( x \to \pm \infty \), \( y \to \infty \)

(2) Graph \( y = x^2 + 5x + c \) on your calculator for the following values of \( c \): \{8, 4, 0, -4, -8\} and sketch. (WINDOW: \( x: [-10, 10], y: [-15, 15] \))

- **What special case occurs at \( c = 0 \)?** The parabola passes through the origin.
- **Check your predictions on your anticipation guide.** Were you correct? Explain why the patterns occur. There are vertical shifts because you are just adding or subtracting a constant to the graph of \( y = x^2 + 5x \), so the \( y \) changes.

(3) Graph \( y = x^2 + bx + 4 \) on your calculator for the following values of \( b \): \{6, 3, 0, -3, -6\} and sketch.

- **What special case occurs at \( b = 0 \)?** The \( y \)-axis is the axis of symmetry and the vertex is at \( (0, 4) \)
- **Check your predictions on your anticipation guide.** Were you correct? Explain why the patterns occur. There are oblique shifts with the \( y \)-intercept remaining the same, but the vertex is becoming more negative because the vertex is affected by \( b \) found using \( \left( -\frac{b}{2a}, f\left( -\frac{b}{2a} \right) \right) \) and \( a \) is 1. The axis of symmetry is \( x = -\frac{b}{2a} \), so when \( b > 0 \), it moves left, and when \( b < 0 \), the axis of symmetry moves right.

Since real zeroes are determined by the discriminant \( b^2 - 4ac \) which in this case is \( b^2 - 16 \), when \( |b| \geq 4 \), there will be real zeroes.

(4) Graph \( y = ax^2 + 5x + 4 \) on your calculator for the following values of \( a \): \{2, 1, 0.5, 0, -0.5, -1, -2\} and sketch.

- **What special case occurs at \( a = 0 \)?** The graph is the line \( y = 5x + 4 \)
- **Check your predictions on your anticipation guide.** Were you correct? Explain why the patterns occur. The \( y \)-intercept remains the same. When \( |a| > 1 \), the parabola is skinny and when \( |a| < 1 \) the parabola is wide. When \( a \) is positive, the parabola opens up; and when \( a \) is negative, the parabola opens down. The axis of symmetry is affected by \( a \), so as \( |a| \) gets bigger, the axis of symmetry approaches \( x = 0 \). Since real zeroes are determined by the discriminant, which in this case is \( 25 - 14a \), when \( |a| \geq \frac{25}{14} \), there will be real zeroes.
What Goes Up:
Position and Time for a Cart on a Ramp

When a cart is given a brief push up a ramp, it will roll back down again after reaching its highest point. Algebraically, the relationship between the position and elapsed time for the cart is quadratic in the general form

\[ y = ax^2 + bx + c \]

where \( y \) represents the position of the cart on the ramp and \( x \) represents the elapsed time. The quantities \( a, b, \) and \( c \) are parameters which depend on such things as the inclination angle of the ramp and the cart’s initial speed. Although the cart moves back and forth in a straight-line path, a plot of its position along the ramp graphed as a function of time is parabolic.

Parabolas have several important points including the vertex (the maximum or minimum point), the \( y \)-intercept (where the function crosses the \( y \)-axis), and the \( x \)-intercepts (where the function crosses the \( x \)-axis). The \( x \)- and \( y \)-intercepts are related to the parameters \( a, b, \) and \( c \) given in the equation above according to the following properties:

1. The \( y \)-intercept is equal to the parameter \( c \).
2. The product of the \( x \)-intercepts is equal to the ratio \( \frac{c}{a} \).
3. The sum of the \( x \)-intercepts is equal to \( -\frac{b}{a} \).

These properties mean that if you know the \( x \)- and \( y \)-intercepts of a parabola, you can find its general equation.

In this activity, you will use a Motion Detector to measure how the position of a cart on a ramp changes with time. When the cart is freely rolling, the position versus time graph will be parabolic, so you can analyze this data in terms of the key locations on the parabolic curve.
Unit 5, Activity 8, Drive the Parabola Lab

Activity 10

OBJECTIVES
- Record position versus time data for a cart rolling up and down a ramp.
- Determine an appropriate parabolic model for the position data using the x- and y-intercept information.

MATERIALS
- TI-83 Plus or TI-84 Plus graphing calculator
- EasyData application
- CBR 2 or Go! Motion and direct calculator cable
- 4-wheeled cart
- board or track at least 1.2 m
- books to support ramp
- Motion Detector and data-collection interface

PROCEDURE
1. Set up the Motion Detector and calculator.
   a. Open the pivoting head of the Motion Detector. If your Motion Detector has a sensitivity switch, set it to Normal as shown.
   b. Turn on the calculator and connect it to the Motion Detector. (This may require the use of a data-collection interface.)
2. Place one or two books beneath one end of the board to make an inclined ramp. The inclination angle should only be a few degrees. Place the Motion Detector at the top of the ramp. Remember that the cart must never get closer than 0.4 m to the detector, so if you have a short ramp you may want to use another object to support the detector.
3. Set up EasyData for data collection.
   a. Start the EasyData application, if it is not already running.
   b. Select File from the Main screen, and then select New to reset the application.
4. So that the zero reference position of the Motion Detector will be about a quarter of the way up the ramp, you will zero the detector while the cart is in this position.
   a. Select Setup from the Main screen, and then select Zero…
   b. Hold the cart still, about a quarter of the way up the ramp. The exact position is not critical, but the cart must be freely rolling through this point in Step 6.
   c. Select Zero to zero the Motion Detector.
5. Practice rolling the cart up the ramp so that you release the cart below the point where you zeroed the detector, and so that the cart never gets closer than 0.4 m to the detector. Be sure to pull your hands away from the cart after it starts moving so the Motion Detector does not detect your hands.
6. Select Start to begin data collection. Wait for about a second, and then roll the cart as you practiced earlier.
7. When data collection is complete, a graph of distance versus time will be displayed. Examine the distance versus time graph. The graph should contain an area of smoothly changing distance. The smoothly changing portion must include two y = 0 crossings.
   Check with your teacher if you are not sure whether you need to repeat the data collection. To repeat data collection, select Main to return to the Main screen and repeat Step 6.
ANALYSIS

1. Since the cart may not have been rolling freely on the ramp the whole time data was collected, you need to remove the data that does not correspond to the free-rolling times. In other words, you only want the portion of the graph that appears parabolic. EasyData allows you to select the region you want using the following steps.

   a. From the distance graph, select \textbf{Anlyz} and then select \textbf{Select Region...} from the menu.
   b. If a warning is displayed on the screen; select to begin the region selection process.
   c. Use the \textbf{and} \textbf{keys to move the cursor to the left edge of the parabolic region and select \textbf{OK} to mark the left bound.}
   d. Use the \textbf{and} \textbf{keys to move the cursor to the right edge of the parabolic region and select \textbf{OK} to select the region.}
   e. Once the calculator finishes performing the selection, you will see the selected portion of the graph filling the width of the screen.

2. Since the cart was not rolling freely when data collection started, adjust the time origin for the graph so that it starts with zero. To do this, you will need to leave EasyData.

   a. Select \textbf{Main} to return to the Main screen.
   b. Exit EasyData by selecting \textbf{Quit} from the Main screen and then selecting \textbf{OK}.

3. To adjust the time origin, subtract the minimum time in the time series from all the values in the series. That will start the time series from zero.

   a. Press \textbf{2nd} \{L1\}.
   b. Press \textbf{−}.
   c. To enter the \textbf{min()} function press \textbf{Math}, use \textbf{to highlight the NUM menu, and press the number adjacent \textbf{min(} to paste the command to the home screen.
   d. Press \textbf{2nd} \{L1\} again and press \) to close the minimum function.
   e. Press \textbf{STO}, and press \textbf{2nd} \{L1\} a third time to complete the expression \textbf{L1 − min(L1)} _ \textbf{L1}.
      Press to perform the calculation.

4. You can find the two \textit{x}-intercepts and the \textit{y}-intercept by tracing across the parabola. Redisplay the graph with the individual points highlighted.

   a. Press \textbf{2nd} \{STAT PLOT\} and press \textbf{ENTER} to select Plot 1.
   b. Change the Plot1 settings to match the screen shown here.
      Press \textbf{ENTER} to select any of the settings you change.
   c. Press \textbf{ZOOM} and then select ZoomStat (use cursor keys to scroll to ZoomStat) to draw a graph with the \textit{x} and \textit{y} ranges set to fill the screen with data.
   d. Press \textbf{TRACE} to determine the coordinates of a point on the graph using the cursor keys.
      Trace across the graph to determine the \textit{y}-intercept along with the first and second \textit{x}-intercepts. You will not be able to get to exact \textit{x}-intercepts because of the discrete points, but choose the points closest to the zero crossing. Round these values to 0.01, and record them in the first Data Table on the Data Collection and Analysis sheet.
Activity 10

5. Determine the product and sum of the \(x\)-intercepts. Record these values in the second DataTable on the Data Collection and Analysis sheet.

6. Use the intercept values, along with the three intercept properties discussed in the introduction, to determine the values of \(a\), \(b\), and \(c\) for the general form parabolic expression \(y = ax^2 + bx + c\). Record these values in the third Data Table.

\[\text{Hint: Write an equation for each of the three properties; then solve this system of equations for } a, b, \text{ and } c.\]

⇒ Answer Question 1 on the Data Collection and Analysis sheet.

7. Now that you have determined the equation for the parabola, plot it along with your data.
   a. Press \(y = \)
   b. Press CLEAR to remove any existing equation.
   c. Enter the equation for the parabola you determined in the \(Y_1\) field. For example, if your equation is \(y = 5x^2 + 4x + 3\), enter \(5\times x^2+4\times x+3\) on the \(Y_1\) line.
   d. Press \(\downarrow\) until the icon to the left of \(Y_1\) is blinking. Press ENTER until a bold diagonal line is shown which will display your model with a thick line
   e. Press GRAPH to see the data with the model graph superimposed.

⇒ Answer Question 2 on the Data Collection and Analysis sheet.

8. You can also determine the parameters of the parabola using the calculator’s quadratic regression function.
   a. Press \(\text{STAT}\) and use the cursor keys to highlight CALC.
   b. Press the number adjacent to QuadReg to copy the command to the home screen.
   c. Press \(\text{2nd} [L1], \text{2nd} [L6]\) to enter the lists containing the data.
   d. Press \(\text{VARS}\) and use the cursor keys to highlight \(Y-VARS\).
   e. Select Function by pressing ENTER
   f. Press ENTER to copy \(Y_1\) to the expression.

On the home screen, you will now see the entry QuadReg \(L_1, L_6, Y_1\). This command will perform a quadratic regression using the \(x\)-values in \(L_1\) and the \(y\)-values in \(L_6\). The resulting regression line will be stored in equation variable \(Y_1\).

   g. Press ENTER to perform the regression.

⇒ Record the regression equation with its parameters in Question 3 on the Data Collection and Analysis sheet.

a. Press GRAPH to see the graph.

⇒ Answer Questions 4-6 on the Data Collection and Analysis sheet.
DATA COLLECTION AND ANALYSIS

DATA TABLES

<table>
<thead>
<tr>
<th></th>
<th>First x intercept</th>
<th>Second x intercept</th>
<th>Product of x intercepts</th>
<th>Sum of x intercepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>y intercept</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| a       |                   |                    |                         |                     |
| b       |                   |                    |                         |                     |
| c       |                   |                    |                         |                     |

QUESTIONS

1. Substitute the values of $a$, $b$, and $c$ you just found into the equation $y = ax^2 + bx + c$. Record the completed modeling equation here.

2. Is your parabola a good fit for the data?

3. Record the regression equation from Step 8 with its parameters.

4. Are the values of $a$, $b$, and $c$ in the quadratic regression equation above consistent with your results from your earlier calculation?

5. In the experiment you just conducted, the vertex on the parabolic distance versus time plot corresponds to a minimum on the graph even though this is the position at which the cart reaches its maximum distance from the starting point along the ramp. Explain why this is so.

6. Suppose that the experiment is repeated, but this time the Motion Detector is placed at the bottom of the ramp instead of at the top. Make a rough sketch of your predicted distance versus time plot for this situation. Discuss how the coefficient $a$ would be affected, if at all.
TEACHER INFORMATION

What Goes Up:
Position and Time for a Cart on a Ramp

1. There are currently four Motion Detectors that can be used for this lab activity. Listed below is the best method for connecting your type of Motion Detector. Optional methods are also included:

   **Vernier Motion Detector**: Connect the Vernier Motion Detector to a CBL 2 or LabPro using the Motion Detector Cable included with this sensor. The CBL 2 or LabPro connects to the calculator using the black unit-to-unit link cable that was included with the CBL 2 or LabPro.

   **CBR**: Connect the CBR directly to the graphing calculator’s I/O port using the extended length I/O cable that comes with the CBR.

   Optionally, the CBR can connect to a CBL 2 or LabPro using a Motion Detector Cable. This cable is not included with the CBR, but can be purchased from Vernier Software & Technology (order code: MDC-BTD).

   **CBR 2**: The CBR 2 includes two cables: an extended length I/O cable and a Calculator USB cable. The I/O cable connects the CBR 2 to the I/O port on any TI graphing calculator. The Calculator USB cable is used to connect the CBR 2 to the USB port located at the top right corner of any TI-84 Plus calculator.

   Optionally, the CBR 2 can connect to a CBL 2 or LabPro using the Motion Detector Cable. This cable is not included with the CBR 2, but can be purchased from Vernier Software & Technology (order code: MDC-BTD).

   **Go! Motion**: This sensor does not include any cables to connect to a graphing calculator. The cable that is included with it is intended for connecting to a computer’s USB port. To connect a Go! Motion to a TI graphing calculator, select one of the options listed below:

   Option I—the Go! Motion connects to a CBL 2 or LabPro using the Motion Detector Cable (order code: MDC-BTD) sold separately by Vernier Software & Technology.

   Option II—the Go! Motion connects to the graphing calculator’s I/O port using an extended length I/O cable (order code: GM-CALC) sold separately by Vernier Software & Technology.

   Option III—the Go! Motion connects to the TI-84 Plus graphing calculator’s USB port using a Calculator USB cable (order code: GM-MINI) sold separately by Vernier Software & Technology.

2. When connecting a CBR 2 or Go! Motion to a TI-84 calculator using USB, the EasyData application automatically launches when the calculator is turned on and at the home screen.
3. A four-wheeled dynamics cart is the best choice for this activity. (Your physics teacher probably has a collection of dynamics carts.) A toy car such as a Hot Wheels or Matchbox car is too small, but a larger, freely-rolling car can be used. A ball can be used, but it is very difficult to have the ball roll directly up and down the ramp. As a result the data quality is strongly dependent on the skill of the experimenter when a ball is used.

4. If a channeled track which forces a ball to roll along a line is used as the ramp, a ball will yield satisfactory data.

5. Note that the ramp angle should only be a few degrees above horizontal. We suggest an angle of five degrees. Most students will create ramps with angles much larger than this, so you might want to have them calculate the angles of their tracks. That will serve both as a trigonometry review and ensure that the ramps are not too steep.

6. It is critical that the student zeroes the Motion Detector in a location that will be crossed by the cart during its roll. If the cart does cross the zero location (both on the way up and the way down), there will be two \( x \)-axis crossings as required by the analysis. If the student does not zero the Motion Detector, or zeroes it in a location that is not crossed by the cart during data collection, then the analysis as presented is not possible.

7. If the experimenter uses care, it is possible to have the cart freely rolling throughout data collection. In this case (as in the sample data below) there is no need to select a region or adjust the time origin, saving several steps.

**SAMPLE RESULTS**

![Raw Data in EasyData](image1)

![Data with parabolic model](image2)

Model equation
DATA TABLES

<table>
<thead>
<tr>
<th>y intercept</th>
<th>First x intercept</th>
<th>Second x intercept</th>
<th>Product of x intercepts</th>
<th>Sum of x intercepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.273</td>
<td>0.40</td>
<td>2.0</td>
<td>.8</td>
<td>2.4</td>
</tr>
</tbody>
</table>

\[
\begin{array}{|c|}
\hline
a & 0.341 \\
\hline
b & -0.818 \\
\hline
c & 0.273 \\
\hline
\end{array}
\]

ANSWERS TO QUESTIONS

1. Model equation is \( y = 0.341x^2 - 0.818x + 0.273 \) (depends on data collected).

2. Model parabola is an excellent fit, as expected since the vertices were taken from the experimental data.

3. Regression quadratic equation is \( y = 0.285 - 0.797x + 0.326x^2 \), or nearly the same as that obtained using the vertex form.

4. The parameters in the calculator’s regression are nearly the same as those determined from the vertex form of the equation.

5. The Motion Detector records distance away from itself. Since the detector was at the top of the ramp, the cart was at its closest (minimum distance) to the detector when the cart was at its highest point.

6. If the experiment were repeated with the Motion Detector at the bottom of the ramp, the distance data would still be parabolic. However, the parabola would open downward, and the coefficient \( a \) would change sign.
In your Bellringer, you found the zeroes and end-behavior of the related graph to help you solve the inequality. What if your equation had only imaginary roots and no real zeroes, how could you use the related graph?

**Quadratic Inequalities** Find the roots and zeroes of the following quadratic equations and fast graph, paying attention only to the x intercepts and the end-behavior. Use the graphs to help you solve the one–variable inequalities by looking at the positive and negative values of y.

(1) Graph $y = x^2 - 3$

<table>
<thead>
<tr>
<th>Zeroes: _______</th>
<th>Roots: ___________</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve for $x$: $x^2 - 3 &gt; 0$</td>
<td></td>
</tr>
</tbody>
</table>

(4) Graph $y = x^2 + 5$

<table>
<thead>
<tr>
<th>Zeroes: _______</th>
<th>Roots: ___________</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve for $x$: $x^2 + 5 &gt; 0$</td>
<td></td>
</tr>
</tbody>
</table>

(2) Graph $y = x^2 + 4x - 6$

<table>
<thead>
<tr>
<th>Zeroes: _______</th>
<th>Roots: ___________</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve for $x$: $x^2 + 4x \leq 6$</td>
<td></td>
</tr>
</tbody>
</table>

(5) Graph $y = -x^2 - 2$

<table>
<thead>
<tr>
<th>Zeroes: _______</th>
<th>Roots: ___________</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve for $x$: $-x^2 - 2 \geq 0$</td>
<td></td>
</tr>
</tbody>
</table>

(3) Graph $y = 4x^2 - 4x + 1$

<table>
<thead>
<tr>
<th>Zeroes: _______</th>
<th>Roots: ___________</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve for $x$: $4x^2 - 4x + 1 &lt; 0$</td>
<td></td>
</tr>
</tbody>
</table>

(6) Graph $y = x^2 - 8x + 20$

<table>
<thead>
<tr>
<th>Zeroes: _______</th>
<th>Roots: ___________</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve for $x$: $x^2 - 8x \leq -20$</td>
<td></td>
</tr>
</tbody>
</table>
In your Bellringer, you found the zeroes and end-behavior of the related graph to help you solve the inequality. What if your equation had only imaginary roots and no real zeroes, how could you use the related graph?

*Answers will vary, but hopefully will talk about end-behavior and the y-values always being positive or always negative, so the solution will be all reals or the empty set.*

**Quadratic Inequalities**

Find the zeroes and roots of the following quadratic equations and fast graph, paying attention only to the x-intercepts and the end-behavior. Use the graphs to help you solve the one-variable inequalities by looking at the positive and negative values of y.

(1) Graph \( y = x^2 - 3 \)
- **zeroes:** \( x = \pm \sqrt{3} \)
- **roots:** \( x = \pm \sqrt{3} \)
- **Solve for:** \( x^2 - 3 > 0 \)
- \( x < -\sqrt{3} \) or \( x > \sqrt{3} \)

(2) Graph \( y = x^2 + 4x - 6 \)
- **zeroes:** \( x = -2 \pm \sqrt{10} \)
- **roots:** \( x = -2 \pm \sqrt{10} \)
- **Solve for:** \( x^2 + 4x \leq 6 \) \( (\text{Hint: isolate 0 first}) \)
- \( -2 - \sqrt{10} \leq x \leq -2 + \sqrt{10} \)

(3) Graph \( y = 4x^2 - 4x + 1 \)
- **zeroes:** \( x = \frac{1}{2} \)
- **roots:** *double root at* \( x = \frac{1}{2} \)
- **Solve for:** \( 4x^2 - 4x + 1 < 0 \)
- empty set

(4) Graph \( y = x^2 + 5 \)
- **zeroes:** none
- **roots:** \( x = \pm i \sqrt{5} \)
- **Solve for:** \( x^2 + 5 > 0 \)
- All real numbers

(5) Graph \( y = -x^2 - 4 \)
- **zeroes:** none
- **roots:** \( x = \pm 2i \)
- **Solve for:** \( -x^2 - 4 \geq 0 \)
- empty set

(6) Graph \( y = x^2 - 8x + 20 \)
- **zeroes:** none
- **roots:** \( x = 4 \pm i \sqrt{6} \)
- **Solve for:** \( x^2 - 8x \leq -20 \)
- empty set
Synthetic Division

(1) \((x^3 + 8x^2 - 5x - 84) \div (x + 5)\)

(a) Use synthetic division to divide and write the answers in equation form as

\[
\frac{\text{dividend}}{\text{divisor}} = \frac{\text{quotient}}{\text{divisor}} + \frac{\text{remainder}}{\text{divisor}}
\]

\[
\frac{x^3 + 8x^2 - 5x - 84}{x + 5} =
\]

(b) Multiply both sides of the equation by the divisor (do not expand) and write in equation form as polynomial = (divisor)(quotient) + remainder) in other words

\[P(x) = (x - c)(Q(x)) + \text{Remainder}\]

\[P(x) = \]

(2) \((x^3 + 8x^2 - 5x - 84) \div (x - 3) \text{ (Same directions as #1)}\)

(a) \[
\frac{x^3 + 8x^2 - 5x - 84}{x - 3} =
\]

(b) \[P(x) = \]

Remainder Theorem

(3) What is the remainder in #1b above? _______ What is c? _____ Find \(P(-5)\). _______

(4) What is the remainder in #2b above? _______ What is c? _____ Find \(P(3)\). _______

(5) Complete the Remainder Theorem: If \(P(x)\) is a polynomial and \(c\) is a number, and if \(P(x)\) is divided by \(x - c\), then ___________

(6) Use your calculators to verify the Remainder Theorem.

(a) Enter \(P(x) = x^3 + 8x^2 - 5x - 84\) into \(y_1\) and find \(P(-5)\) and \(P(3)\) on the home screen as \(y_1(-5)\) and \(y_1(3)\).

(b) Practice: \(f(x) = 4x^3 - 6x^2 + 2x - 5\). Find \(f(3)\) using synthetic division and verify on the calculator.

(c) Explain why synthetic division is sometimes called synthetic substitution.
Unit 5, Activity 12, Factor Theorem Discovery Worksheet

Factor Theorem

(7) Define factor $\equiv$ ________________________________

(8) Factor the following:
   (a) 12  (b) $x^2 - 9$  (c) $x^2 - 5$  (d) $x^2 + 4$

   (e) $x^3 + 8x^2 - 5x - 84$ (Hint: See #2b above.) = ________________________________

(9) Using 8(e) complete the Factor Theorem: If $P(x)$ is a polynomial, then $x - c$ is a factor of $P(x)$ if and only if ________________________________

(10) Work the following problem to verify the Factor Theorem: Factor $f(x) = x^2 + 3x + 2$ and find $f(-2)$ and $f(-1)$.

(11) In #1 and #2 above you redefined the division problem as $P(x) = (x - c)(Q(x)) + \text{Remainder}$. $Q(x)$ is called a depressed polynomial because the powers of $x$ are one less than the powers of $P(x)$. The goal is to develop a quadratic depressed equation that can be solved by quadratic function methods.

   (a) In #2b, you rewrote $\frac{x^3 + 8x^2 - 5x - 84}{x - 3} = x^2 - 11x + 28$ and $P(x) = (x^2 + 11x + 28)(x-3)$

      What is the depressed equation? ________________________________

   (b) Finish factoring $x^3 + 8x^2 - 5x - 84 = ________________________________$

      List all the zeroes: ________________________________

(12) (a) Use synthetic division to determine if $(x - 2)$ is a factor of $y = x^3 + 2x^2 - 5x - 6$.

   (b) What is the depressed equation? ________________________________

   (c) Factor $y$ completely: ________________________________
**Unit 5, Activity 12, Factor Theorem Discovery Worksheet**

**Factor Theorem Practice**

Given one factor of the polynomial, use synthetic division and the depressed polynomial to factor completely.

(1a) \(x + 1; x^3 + x^2 - 16x - 16\), \hspace{1cm} (1b) \(x + 6; x^3 + 7x^2 - 36\).

Given one factor of the polynomial, use synthetic division to find all the roots of the equation.

(2a) \(x - 1; x^3 - x^2 - 2x + 2 = 0\), \hspace{1cm} (2b) \(x + 2; x^3 - x^2 - 2x + 8 = 0\).

Given two factors of the polynomial, use synthetic division and the depressed polynomials to factor completely. (*Hint: Use the second factor in the 3\(^{rd}\) degree depressed polynomial to get a depressed quadratic polynomial, then factor.*)

(3a) \(x - 1, x - 3; x^4 - 10x^3 + 35x^2 - 50x + 24\) \hspace{1cm} (3b) \(x + 3, x - 4, x^4 - 2x^3 - 13x^2 + 14x + 24\).
Unit 5, Activity 12, Factor Theorem Discovery Worksheet with Answers

Synthetic Division

(1) \((x^3 + 8x^2 - 5x - 84) \div (x + 5)\)

(a) Use synthetic division to divide and write the answers in equation form as
\[
\frac{\text{dividend}}{\text{divisor}} = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}
\]
\[
\frac{x^3 + 8x^2 - 5x - 84}{x + 5} = x^2 + 3x - 20 + \frac{16}{x + 5}
\]

(b) Multiply both sides of the equation by the divisor \((do not expand)\) and write in equation form as polynomial \(= (\text{divisor})(\text{quotient}) + \text{remainder}\) in other words
\[P(x) = (x - c)(Q(x)) + \text{Remainder}
\]
\[P(x) = (x + 5)(x^2 + 3x - 20) + 16
\]

(2) \((x^3 + 8x^2 - 5x - 84) \div (x - 3)\) (Same directions as #1)

(a) \[
\frac{x^3 + 8x^2 - 5x - 84}{x - 3} = x^2 + 11x + 28 + \frac{0}{x - 3}
\]

(b) \[P(x) = (x - 3)(x^2 + 11x + 28) + 0
\]

Remainder Theorem

(3) What is the remainder in #1b above? \(16\) What is \(c\)? \(-5\) Find \(P(-5)\). \(16\).

(4) What is the remainder in #2b above? \(0\) What is \(c\)? \(3\) Find \(P(3)\). \(0\).

(5) Complete the Remainder Theorem: If \(P(x)\) is a polynomial and \(c\) is a number, and if \(P(x)\) is divided by \(x - c\), then the remainder equals \(P(c)\).

(6) Use your calculators to verify the Remainder Theorem.
(a) Enter \(P(x) = x^3 + 8x^2 - 5x - 84\) into \(y_1\) and find \(P(-5)\) and \(P(3)\) on the home screen as \(y_1(-5)\) and \(y_1(3)\).
(b) Practice: \(f(x) = 4x^3 - 6x^2 + 2x - 5\). Find \(f(3)\) using synthetic division and verify on the calculator.
\[
\begin{array}{c|cccc}
3 & 4 & -6 & 2 & -5 \\
\hline
& 12 & 18 & 60 \\
4 & 6 & 20 & 55
\end{array}
\]
\[f(3) = 4(3)^3 - 6(3)^2 + 2(3) - 5 = 55
\]

(c) Explain why synthetic division is sometimes called \textit{synthetic substitution}. \textit{See 6(b)}
Unit 5, Activity 12, Factor Theorem Discovery Worksheet with Answers

Factor Theorem

(7) Define factor ≡ two or more numbers or polynomials that are multiplied together to get a third number or polynomial.

(8) Factor the following:
   (a) 12  (b) \(x^2 - 9\)  (c) \(x^2 - 5\)  (d) \(x^2 + 4\)

   \[
   12 = (3)(4) \quad (x - 3)(x + 3) \quad \left(x - \sqrt{5}\right)\left(x + \sqrt{5}\right) \quad (x + 2i)(x - 2i)
   \]

   (e) \(x^3 + 8x^2 - 5x - 84\) (Hint: See #2b above.) = \((x - 3)(x^2 + 11x + 28)\)

(9) Using 8(e) complete the Factor Theorem: If \(P(x)\) is a polynomial, then \(x - c\) is a factor of \(P(x)\) if and only if \(P(c) = 0\). (The remainder is 0 therefore \(P(c)\) must be 0.)

(10) Work the following problem to verify the Factor Theorem: Factor \(f(x) = x^2 + 3x + 2\) and find \(f(-2)\) and \(f(-1)\).

   \[
   x^2 + 3x + 2 = (x + 2)(x + 1) \quad f(-2) = 0, f(-1) = 0
   \]

(11) In #1 and #2 above you redefined the division problem as \(P(x) = (x - c)(Q(x)) + \text{Remainder}\). \(Q(x)\) is called a depressed polynomial because the powers of \(x\) are one less than the powers of \(P(x)\). The goal is to develop a quadratic depressed equation that can be solved by quadratic function methods.

   (a) In #2b, you rewrote \(\frac{x^3 + 8x^2 - 5x - 84}{x - 3} = x^2 + 11x + 28\) and \(P(x) = (x^2 + 11x + 28)(x - 3)\)

   What is the depressed equation? \(C(x) = x^2 - 11x + 28\)

   (b) Finish factoring \(x^3 + 8x^2 - 5x - 84 = (x - 3)(x - 7)(x - 4)\)

   List all the zeroes: \(\{3, 7, 4\}\)

(12) (a) Use synthetic division to determine if \((x - 2)\) is a factor of \(y = x^3 + 2x^2 - 5x - 6\).

   \[
   \begin{array}{c|cccc}
   2 & 1 & 2 & -5 & -6 \\
   & & 2 & 8 & 6 & \hline
   1 & 4 & 3 & 0
   \end{array}
   \]

   Yes, \((x - 2)\) is a factor.

   (b) What is the depressed equation? \(x^2 + 4x + 3\)

   (c) Factor \(y\) completely: \((x - 2)(x + 3)(x + 1)\)
Factor Theorem Practice

Given one factor of the polynomial, use synthetic division and the depressed polynomial to factor completely.

(1a) \(x + 1; x^3 + x^2 − 16x − 16\), \((x + 1)(x − 4)(x + 4)\)  
(1b) \(x + 6; x^3 + 7x^2 − 36\), \((x+6)(x + 3)(x − 2)\)

Given one factor of the polynomial, use synthetic division to find all the roots of the equation.

(2a) \(x − 1; x^3 − x^2 − 2x + 2 = 0\), \(\{1, \sqrt{2}, -\sqrt{2}\}\)  
(2b) \(x + 2; x^3 − x^2 − 2x + 8 = 0\), \(\left\{-2, \frac{3}{2} + \frac{\sqrt{5}}{2}i, \frac{3}{2} − \frac{\sqrt{5}}{2}i\right\}\)

Given two factors of the polynomial, use synthetic division and the depressed polynomials to factor completely. (Hint: Use the second factor in the 3rd degree depressed polynomial to get a depressed quadratic polynomial, then factor.)

(3a) \(x − 1, x − 3; x^4 − 10x^3 + 35x^2 − 50x + 24\), \((x−1)(x−3)(x−2)(x−4)\)  
(3b) \(x + 3, x − 4, x^4 − 2x^3 − 13x^2 + 14x + 24\), \((x+3)(x−4)(x−2)(x+1)\)
Unit 5, Activity 13, Exactly Zero

Graph the following on your calculator and find all exact zeroes and roots and factors:

(1) \( f(x) = x^3 + 2x^2 - 10x + 4 \)

(2) \( f(x) = x^4 + 2x^3 - 4x^2 - 6x + 3 \)

(3) \( f(x) = x^4 + 8x^3 - 22x^2 - 48x + 96 \)

(4) \( f(x) = 2x^3 + 7x^2 - x - 2 \)  (Hint: Leading coefficient is 2; therefore, factors must multiply out to get that coefficient)

(5) \( f(x) = 3x^3 - 4x^2 - 28x - 16 \)

(6) Discuss the process used to find the exact answers.
Graph the following on your calculator and find all exact zeroes and roots and factors:

(1) \( f(x) = x^3 + 2x^2 - 10x + 4 \)
zeroes/roots: \( \{2, 2 + \sqrt{6}, 2 - \sqrt{6}\} \), factors: \( f(x) = (x - 2)(x - 2 - \sqrt{6})(x - 2 + \sqrt{6}) \)

(2) \( f(x) = x^4 + 2x^3 - 4x^2 - 6x + 3 \)
zeroes/roots: \( \{3, -3, -1 + \sqrt{2}, -1 - \sqrt{2}\} \),
factors: \( f(x) = (x - 3)(x + 3)(x + 1 - \sqrt{2})(x + 1 + \sqrt{2}) \)

(3) \( f(x) = x^4 + 8x^3 - 22x^2 - 48x + 96 \)
zeroes: \( x = -4 \), roots: \( \{ -4, -4\sqrt{2}, -4i\sqrt{2} \} \),
factors: \( f(x) = (x + 4)^2(x - i\sqrt{2})(x + i\sqrt{2}) \)

(4) \( f(x) = 2x^3 + 7x^2 - x - 2 \) (Hint: Leading coefficient is 2; therefore, factors must multiply out to get that coefficient)
zeroes/roots: \( \left\{ -\frac{1}{2}, -\frac{3}{2} + \frac{\sqrt{17}}{2}, -\frac{3}{2} - \frac{\sqrt{17}}{2} \right\} \),
factors: \( f(x) = (2x + 1) \left( x - \frac{-3 + \sqrt{17}}{2} \right) \left( x - \frac{-3 - \sqrt{17}}{2} \right) \)

(5) \( f(x) = 3x^3 - 4x^2 - 28x - 16 \)
zeroes/roots: \( \{4, -2, \frac{2}{3}\} \), factors: \( f(x) = (3x - 2)(x - 4)(x + 2) \)

(6) Discuss the process used to find the exact answers. Find all rational roots on the calculator. Use these with synthetic division to find a depressed quadratic equation and solve with the quadratic formula.
Unit 5, Activity 14, Rational Roots of Polynomials

Name_________________________________________ Date_____________________

Vocabulary Self–Awareness Chart

(1) Rate your understanding of each number system with either a “+” (understand well), a “✓”
(limited understanding or unsure), or a “−” (don’t know)

<table>
<thead>
<tr>
<th>Complex Number System Terms</th>
<th>+</th>
<th>✓</th>
<th>−</th>
<th>Roots from Exact Zero BLM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 integer</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 rational number</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 irrational number</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 real number</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 imaginary number</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 complex number</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(2) List all of the roots found in the Exact Zero BLM completed in Activity #13.

(1) \( f(x) = x^3 + 2x^2 - 10x + 4 \)
(2) \( f(x) = x^4 + 2x^3 - 4x^2 - 6x + 3 \)
(3) \( f(x) = x^4 + 8x^3 - 22x^2 - 48x + 96 \)
(4) \( f(x) = 2x^3 + 7x^2 - x - 2 \)
(5) \( f(x) = 3x^3 - 4x^2 - 28x - 16 \)

(3) Fill the roots in the chart in the proper classification.

Rational Root Theorem

(4) Define rational number: __________________________________________________________

(5) Circle all the rational roots in the equations above.
(6) What is alike about all the polynomials that have integer rational roots?
(7) What is alike about all the polynomials that have fraction rational roots?
(8) Complete the Rational Root Theorem: If a polynomial has integral coefficients, then any
rational roots will be in the form \( \frac{p}{q} \) where \( p \) is ________________________________

and \( q \) is ________________________________
(9) Identify the $p = \text{constant}$ and the $q = \text{leading coefficient}$ of the following equations from the Exact Zero BLM and list all possible rational roots:

<table>
<thead>
<tr>
<th>polynomial</th>
<th>factors of $p$</th>
<th>factors of $q$</th>
<th>possible rational roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $f(x) = x^3 + 2x^2 - 10x + 4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) $f(x) = x^4 + 2x^3 - 4x^2 - 6x + 3$</td>
<td></td>
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<td></td>
</tr>
<tr>
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</table>

**Additional Theorems for Graphing Aids**

(10) Fundamental Theorem of Algebra: Every polynomial function with complex coefficients has at least one root in the set of complex numbers.

(11) Number of Roots Theorem: Every polynomial function of degree $n$ has exactly $n$ complex roots. (Some may have multiplicity.)

(12) Complex Conjugate Root Theorem: If a complex number $a + bi$ is a solution of a polynomial equation with real coefficients, then the conjugate $a - bi$ is also a solution of the equation. (e.g. If $2 + 3i$ is a root then $2 - 3i$ is a root.)

(13) Intermediate Value Theorem for Polynomials: (as applied to locating zeroes). If $f(x)$ defines a polynomial function with real coefficients, and if for real numbers $a$ and $b$ the values of $f(a)$ and $f(b)$ are opposite signs, then there exists at least one real zero between $a$ and $b$.

(a) Consider the following chart of values for a polynomial. Because a polynomial is continuous, in what intervals of $x$ does the Intermediate Values Theorem guarantee a zero?

<table>
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<tr>
<th>$x$</th>
<th>$-5$</th>
<th>$-4$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
<th>$4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>$-1482$</td>
<td>$-341$</td>
<td>$216$</td>
<td>$357$</td>
<td>$250$</td>
<td>$63$</td>
<td>$-36$</td>
<td>$121$</td>
<td>$702$</td>
<td>$1875$</td>
</tr>
</tbody>
</table>

This is data for the polynomial $f(x) = 28x^3 + 68x^2 - 83x - 63$.

(b) List all the possible rational roots:

(c) Circle the ones that lie in the Interval of Zeroes.

(d) Use synthetic division with the circled possible rational roots to find a depressed equation to locate the remaining roots.
Vocabulary Self–Awareness Chart

(1) Rate your understanding of each number system with either a “+” (understand well), a “−” (limited understanding or unsure), or a “−” (don’t know)

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<td></td>
<td></td>
<td>2, −2, 3, −3, 4, −4</td>
</tr>
<tr>
<td>2 rational number</td>
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<td></td>
<td></td>
<td>2, −2, 3, −3, 4, −4, $\frac{1}{2}$, $\frac{2}{3}$</td>
</tr>
<tr>
<td>3 irrational number</td>
<td></td>
<td></td>
<td></td>
<td>2 + $\sqrt{6}$, $2 - \sqrt{6}$, $-1 + \sqrt{2}$, $-1 - \sqrt{2}$, $-\frac{3}{2}$, $\frac{\sqrt{17}}{2}$, $\frac{-3}{\sqrt{2}}$, $\frac{-\sqrt{17}}{2}$</td>
</tr>
<tr>
<td>4 real number</td>
<td></td>
<td></td>
<td></td>
<td>all the answers in #1–3 above</td>
</tr>
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<td>5 imaginary number</td>
<td></td>
<td></td>
<td></td>
<td>$i\sqrt{6}$, $-i\sqrt{6}$</td>
</tr>
<tr>
<td>6 complex number</td>
<td></td>
<td></td>
<td></td>
<td>all the answers in #1–5 above</td>
</tr>
</tbody>
</table>

(2) List all of the roots found in the Exact Zero BLM completed in Activity #13.

(1) $f(x) = x^3 + 2x^2 − 10x + 4 \quad \{2, 2 + \sqrt{6}, 2 - \sqrt{6}\}$

(2) $f(x) = x^4 + 2x^3 − 4x^2 − 6x + 3 \quad \{3, -3, -1 + \sqrt{2}, -1 - \sqrt{2}\}$

(3) $f(x) = x^4 + 8x^3 − 22x^2 − 48x + 96 \quad \{-4, -4, i\sqrt{6}, -i\sqrt{6}\}$

(4) $f(x) = 2x^3 + 7x^2 − x − 2 \quad \{-\frac{1}{2}, -\frac{3}{2} + \frac{\sqrt{17}}{2}, -\frac{3}{2} - \frac{\sqrt{17}}{2}\}$

(5) $f(x) = 3x^3 − 4x^2 − 28x − 16 \quad \{4, -2, \frac{2}{3}\}$

(3) Fill the roots in the chart in the proper classification.

Rational Root Theorem

(4) Define rational number: $\frac{p}{q}$ where $p$ and $q$ are integers and $q \neq 0$. All terminating and repeating decimals can be expressed as fractions in this form

(5) Circle all the rational roots in the equations above.

(6) What is alike about all the polynomials that have integer rational roots? The leading coefficient = 1

(7) What is alike about all the polynomials that have fraction rational roots? The leading coefficient $\neq 1$

(8) Complete the Rational Root Theorem: If a polynomial has integral coefficients, then any rational roots will be in the form $\frac{p}{q}$ where $p$ is a factor of the constant and $q$ is a factor of the leading coefficient.
(9) Identify the $p =$ constant and the $q =$ leading coefficient of the following equations from the Exact Zero BLM and list all possible rational roots:

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<tbody>
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<td>$\pm 1$</td>
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<td>$\pm 1$</td>
<td>$\pm 1, \pm 3$</td>
</tr>
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<tr>
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<td>$\pm 1, \pm 2$</td>
<td>$\pm 1, \pm 2$</td>
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<td>$\pm 1, \pm 3$</td>
<td>$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 1/3, \pm 2/3, \pm 4/3, \pm 8/3, \pm 16/3$</td>
</tr>
</tbody>
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(13) Intermediate Value Theorem for Polynomials: (as applied to locating zeroes). If $f(x)$ defines a polynomial function with real coefficients, and if for real numbers $a$ and $b$ the values of $f(a)$ and $f(b)$ are opposite signs, then there exists at least one real zero between $a$ and $b$.

(a) Consider the following chart of values for a polynomial. Because a polynomial is continuous, in what intervals of $x$ does the Intermediate Values Theorem guarantee a zero?

Intervals of Zeros: $(-4, -3), (0, 1), (1, 2)$

<table>
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<tr>
<th>$x$</th>
<th>$-5$</th>
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<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
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<td>$1875$</td>
</tr>
</tbody>
</table>

This is data for the polynomial $f(x) = 28x^3 + 68x^2 - 83x - 63$.

(b) List all the possible rational roots:

Factors of 63: $\{\pm 1, \pm 3, \pm 7, \pm 9, \pm 21, \pm 63\}$, factors of 28: $\{\pm 1, \pm 2, \pm 4, \pm 7, \pm 14, \pm 28\}$

Possible rational roots:

$\pm \left\{ \frac{1}{2}, \frac{1}{3}, \frac{1}{7}, \frac{1}{14}, \frac{1}{28}, \frac{1}{4}, \frac{1}{2}, \frac{1}{3}, \frac{1}{14}, \frac{1}{28}, \frac{2}{3}, \frac{3}{7}, \frac{3}{14}, \frac{3}{28}, \frac{4}{7}, \frac{4}{14}, \frac{9}{7}, \frac{9}{14}, \frac{21}{14}, \frac{21}{7}, \frac{63}{14}, \frac{63}{7} \right\}$

(c) Circle the ones that lie in the Interval of Zeros.

$\left\{ \frac{1}{2}, \frac{1}{3}, \frac{1}{7}, \frac{1}{14}, \frac{1}{28}, \frac{1}{4}, \frac{1}{2}, \frac{1}{3}, \frac{1}{14}, \frac{1}{28}, \frac{2}{3}, \frac{3}{7}, \frac{3}{14}, \frac{3}{28}, \frac{4}{7}, \frac{4}{14}, \frac{9}{7}, \frac{9}{14}, \frac{21}{14}, \frac{21}{7}, \frac{63}{14}, \frac{63}{7} \right\}$

Try $\frac{7}{2}$ first because it is the only one in that interval.

(d) Use synthetic division with the circled possible rational roots to find a depressed equation to locate the remaining roots.

$\left\{ \frac{1}{2}, \frac{1}{3}, \frac{1}{7}, \frac{1}{14}, \frac{1}{28}, \frac{1}{4}, \frac{1}{2}, \frac{1}{3}, \frac{1}{14}, \frac{1}{28}, \frac{2}{3}, \frac{3}{7}, \frac{3}{14}, \frac{3}{28}, \frac{4}{7}, \frac{4}{14}, \frac{9}{7}, \frac{9}{14}, \frac{21}{14}, \frac{21}{7}, \frac{63}{14}, \frac{63}{7} \right\}$
Unit 5, Activity 15, Solving the Polynomial Mystery

Answer #1 – 8 below concerning this polynomial:

\[ f(x) = 4x^4 - 4x^3 - 11x^2 + 12x - 3 \]

(1) How many roots does the Fundamental Theorem of Algebra guarantee this equation has? ___

(2) How many roots does the Number of Roots Theorem say this equation has? _______

(3) List all the possible rational roots: ____________________________________________

(4) Use the chart below and the Intermediate Value Theorem to locate the interval/s of the zeroes.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1.5</th>
<th>-1</th>
<th>-0.5</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>25</td>
<td>-12</td>
<td>-18</td>
<td>-11</td>
<td>-3</td>
<td>0</td>
<td>-2</td>
<td>-3</td>
<td>9</td>
<td>52</td>
<td>150</td>
</tr>
</tbody>
</table>

(5) If you have one root, use synthetic division to find the depressed equation and rewrite \( y \) as a factored equation with one binomial root and the depressed equation.

\[ y = \left( \frac{\text{factor}}{\text{depressed equation}} \right) \]

(6) Use synthetic division on the depressed equation to find all the other roots. List all the roots repeating any roots that have multiplicity. {__________________________}

(7) Write the equation factored with no fractions and no exponents greater than one.
(8) Graph \( f(x) \) without a calculator using all the available information in questions #1–7 on the previous page.
Answer all the questions on this page concerning this polynomial:

\[ f(x) = 4x^4 - 4x^3 - 11x^2 + 12x - 3 \]

(1) How many roots does the Fundamental Theorem of Algebra guarantee this equation has? __1__

(2) How many roots does the Number of Roots Theorem say this equation has? __4__

(3) List all the possible rational roots:

\[ \pm \frac{1}{2} \pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4} \]

(4) Use the chart at below and the Intermediate Value Theorem to locate the interval/s of the zeroes.

\[ (-2, -\frac{3}{2}), \text{ root at } x = \frac{1}{2}, (\frac{3}{2}, 2) \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-\frac{3}{2}</th>
<th>-1</th>
<th>-\frac{1}{2}</th>
<th>0</th>
<th>\frac{1}{2}</th>
<th>1</th>
<th>\frac{3}{2}</th>
<th>2</th>
<th>\frac{5}{2}</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>25</td>
<td>-12</td>
<td>-18</td>
<td>-11</td>
<td>-3</td>
<td>0</td>
<td>-2</td>
<td>-3</td>
<td>9</td>
<td>52</td>
<td>150</td>
</tr>
</tbody>
</table>

(5) If you have one root, use synthetic division to find the depressed equation and rewrite \( y \) as a factored equation with one binomial root and the depressed equation.

\[ y = (x - \frac{1}{2}) \left( 4x^3 - 2x^2 - 12x + 6 \right) \]

\[ \text{depressed equation} \]

\[ \frac{1}{2} \]

\[ 4 \quad -4 \quad -11 \quad 12 \quad -3 \]

\[ 2 \quad -1 \quad -6 \quad 3 \]

\[ 4 \quad -2 \quad -12 \quad 6 \quad 0 \]

\( \frac{1}{2} \text{ may be a double root so try it again} \)

(6) Use synthetic division on the depressed equation to find all the other roots.

List all the roots repeating any roots that have multiplicity. \{ \frac{1}{2}, \frac{1}{2}, \sqrt{3}, -\sqrt{3} \}

\[ \frac{1}{2} \]

\[ 4 \quad -2 \quad -12 \quad 6 \]

\[ 2 \quad 0 \quad -6 \]

\[ 4 \quad 0 \quad -12 \quad 0 \]

\[ \text{Depressed equation: } 4x^2 - 12 = 0 \]

\[ 4x^2 = 12 \]

\[ x^2 = 3 \quad \Rightarrow \quad x = \pm \sqrt{3} \]

(7) Write the equation factored with no fractions and no exponents greater than one.

\[ y = (2x - 1)(2x - 1)(x - \sqrt{3})(x + \sqrt{3}) \]
(8) Graph \( f(x) \) without a calculator using all the available information in questions #1–7 on the previous page.
6.1 **Laws of Exponents** – write rules for adding, subtracting, multiplying and dividing values with exponents, raising an exponent to a power, and using negative and fractional exponents.

6.2 **Solving Exponential Equations** – write the rules for solving two types of exponential equations: same base and different bases (e.g., solve \(2^x = 8^{x-1}\) without calculator; solve \(2^x = 3^{x-1}\) with and without calculator).

6.3 **Exponential Function with Base** \(a\) – write the definition, give examples of graphs with \(a > 1\) and \(0 < a < 1\), and locate three ordered pairs, give the domains, ranges, intercepts, and asymptotes for each.

6.4 **Exponential Regression Equation** - give a set of data and explain how to use the method of finite differences to determine if it is best modeled with an exponential equation, and explain how to find the regression equation.

6.5 **Exponential Function Base** \(e\) – define \(e\), graph \(y = e^x\) and then locate 3 ordered pairs, and give the domain, range, asymptote, intercepts.

6.6 **Compound Interest Formula** – define continuous and finite, explain and give an example of each symbol.

6.7 **Inverse Functions** – write the definition, explain one-to-one correspondence, give an example to show the test to determine when two functions are inverses, graph the inverse of a function, find the line of symmetry and the domain and range, explain how to find inverse analytically and how to draw an inverse on calculator.

6.8 **Logarithm** – write the definition and explain the symbols used, define common logs, characteristic, and mantissa, and list the properties of logarithms.

6.9 **Laws of Logs and Change of Base Formula** – list the laws and the change of base formula and give examples of each.

6.10 **Solving Logarithmic Equations** – explain rules for solving equations, identify the domain for an equation, find \(\log_2 8\) and \(\log_5 125\), and solve each of these equations for \(x\): \(\log x = 2\), \(\log_ax = 2\), \(\log_{(x-3)}+\log_{x^2}=1\).

6.11 **Logarithmic Function Base** \(a\) – write the definition, graph \(y = \log_ax\) with \(a < 1\) and \(a > 1\) and locate three ordered pairs, identify the domain, range, intercepts, and asymptotes, and find domain of \(y = \log(x^2+7x+ 10)\).

6.12 **Natural Logarithm Function** – write the definition and give the approximate value of \(e\), graph \(y = \ln x\) and give the domain, range, and asymptote, and locate three ordered pairs, solve \(\ln x = 2\) for \(x\).

6.13 **Exponential Growth and Decay** – define half-life and solve an example problem, give and solve an example of population growth using \(Pe^{rt}\).
Simplify and explain in words the law of exponents used:

1) \(a^2a^3 = \)
   \[a^5 = \]

2) \(b^7 = \)
   \[b^3 = \]

3) \((c^3)^4 = \)

4) \(2x^5 + 3x^5 = \)

5) \((2x)^3 = \)

6) \((a + b)^2 = \)

7) \(x^0 = \)

8) \(2^{-1} = \)
# Exponential Graph Transformations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Sketch</th>
<th>Domain</th>
<th>Range</th>
<th>x–intercept</th>
<th>y–intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = 2^x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = 3^x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = 5^x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = \frac{1}{2}^x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = \frac{1}{5}^x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = \frac{1}{8}^x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = -2^x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = -\frac{1}{2}^x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Analysis of Exponential Graphs

Answer the following questions concerning the graphs of exponential functions:

1) What point do most of the graphs have in common? _______________________________

2) Which ones do not have that point in common and what is different about them? ______

3) What is the effect of putting a negative sign in front of $b^x$? __________________________

4) What happens to the graph as $b$ increases in problems 1, 2, and 3? __________________

5) Describe the graph if $b = 1$. ___________________________________________________

6) Describe the difference in graphs for problems 1 through 6 if $b > 1$ and if $0 < b < 1$.

_________________________________________________________________________________

7) What is the domain of all the graphs? _____________________________________________

8) What is the range of the graphs? _________________________________________________

9) Are there any asymptotes? If so, what is the equation of the asymptote? ________

10) Predict what the graph of $y = 2^{(x-3)} + 4$ would look like before you graph on your calculator and explain why.

Graph without a calculator and check yourself on the calculator. What is the new $y$–intercept and asymptote?

_________________________________________________________________________________

11) Predict what the graph of $y = 2^{-x}$ would look like before you graph on your calculator and explain why.

Graph without a calculator and check yourself on the calculator. Is it similar to any of the previous graphs and why?
**Unit 6, Activity 2, Graphing Exponential Functions Discovery Worksheet with Answers**

Name ________________________  Key ________________________  Date ________________________

**Exponential Graph Transformations**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Sketch</th>
<th>Domain</th>
<th>Range</th>
<th>$x$-intercept</th>
<th>$y$-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 $f(x) = 2^x$</td>
<td><img src="image1" alt="Sketch" /></td>
<td>all reals</td>
<td>$y &gt; 0$</td>
<td>none</td>
<td>$(0, 1)$</td>
</tr>
<tr>
<td>2 $f(x) = 3^x$</td>
<td><img src="image2" alt="Sketch" /></td>
<td>all reals</td>
<td>$y &gt; 0$</td>
<td>none</td>
<td>$(0, 1)$</td>
</tr>
<tr>
<td>3 $f(x) = 5^x$</td>
<td><img src="image3" alt="Sketch" /></td>
<td>all reals</td>
<td>$y &gt; 0$</td>
<td>none</td>
<td>$(0, 1)$</td>
</tr>
<tr>
<td>4 $f(x) = \frac{1}{2}^x$</td>
<td><img src="image4" alt="Sketch" /></td>
<td>all reals</td>
<td>$y &gt; 0$</td>
<td>none</td>
<td>$(0, 1)$</td>
</tr>
<tr>
<td>5 $f(x) = \frac{1}{5}^x$</td>
<td><img src="image5" alt="Sketch" /></td>
<td>all reals</td>
<td>$y &gt; 0$</td>
<td>none</td>
<td>$(0, 1)$</td>
</tr>
<tr>
<td>6 $f(x) = \frac{1}{8}^x$</td>
<td><img src="image6" alt="Sketch" /></td>
<td>all reals</td>
<td>$y &gt; 0$</td>
<td>none</td>
<td>$(0, 1)$</td>
</tr>
<tr>
<td>7 $f(x) = -2^x$</td>
<td><img src="image7" alt="Sketch" /></td>
<td>all reals</td>
<td>$y &lt; 0$</td>
<td>none</td>
<td>$(0, -1)$</td>
</tr>
<tr>
<td>8 $f(x) = -\frac{1}{2}^x$</td>
<td><img src="image8" alt="Sketch" /></td>
<td>all reals</td>
<td>$y &lt; 0$</td>
<td>none</td>
<td>$(0, -1)$</td>
</tr>
</tbody>
</table>
**Analysis of Exponential Graphs**

Answer the following questions concerning the graphs of exponential functions:

1) What point do most of the graphs have in common?  **y-intercept (0,1)**

2) Which ones do not have that point in common and what is different about them?  **#7 and 8, negative leading coefficient**

3) What is the effect of putting a negative sign in front of \( b \)?  **rotate on the x-axis**

4) What happens to the graph as \( b \) increases in problems 1, 2, and 3?  **it gets steeper**

5) Describe the graph if \( b = 1 \).  **horizontal line at y = 1**

6) Describe the difference in graphs for #1 through #6 if \( b > 1 \) and if \( 0 < b < 1 \).

   **If \( b > 1 \) then the end-behavior as \( x \) approaches \( \infty \) is \( \infty \) and as \( x \) approaches \( -\infty \) is 0.**

   **If \( b < 1 \), then it has the opposite end-behavior**

7) What is the domain of all the graphs?  **all real numbers**

8) What is the range of the graphs?  **#1–6 \( y > 0 \), #7 and 8 \( y < 0 \)**

9) Are there any asymptotes? If so, what is the equation of the asymptote?  **\( y = 0 \)**

10) Predict what the graph of \( y = 2^{(x-3)} + 4 \) would look like before you graph on your calculator and explain why.

   **Shift right 3 and up 4**

   Graph without a calculator and check yourself on the calculator. What is the new \( y \)-intercept and asymptote?

   **\( y \)-intercept: (0, 4.125) Asymptote: \( y = 4 \)**

11) Predict what the graph of \( y = 2^{-x} \) would look like before you graph on your calculator and explain why.

   **rotate \( y = 2^x \) on the \( y \)-axis**

   Graph without a calculator and check yourself on the calculator. Is it similar to any of the previous graphs and why?  **Similar to \( f(x) = \frac{1}{2^x} \) because**

   \[
   2^{-x} = \frac{1}{2^x} = \left( \frac{1}{2} \right)^x
   \]
Unit 6, Activity 3, Exponential Regression Equations

Real World Exponential Data

Enter the following data into your calculator:

To enter data on a TI 84 calculator: STAT 1:Edit, enter data into L1 and L2. To set up the plot of the data: 2ND [STAT PLOT] (above Y=), 1:PLOT1, ENTER, On, Type: Xlist: L1, Ylist: L2, Mark (any). To graph the scatter plot: ZOOM 9: ZoomStat

Wind tunnel experiments are used to test the wind friction or resistance of an automobile at the following speeds.

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>Resistance (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>6.4</td>
</tr>
<tr>
<td>21</td>
<td>9.2</td>
</tr>
<tr>
<td>34</td>
<td>17.0</td>
</tr>
<tr>
<td>40</td>
<td>22.4</td>
</tr>
<tr>
<td>45</td>
<td>30.2</td>
</tr>
<tr>
<td>55</td>
<td>59.2</td>
</tr>
</tbody>
</table>

Determine a regression equation for the data changing the constants $A$ and $B$ in $f(x) = AB^x$ to find the best equation to fit the data. Change the individual constants until your graph matches the data or use the Transformation APPS. Do not use the regression feature of the calculator. Then enter your equation to see if it matches the data.

(1) Write your equation and discuss why you chose the values $A$ and $B$.

(2) When each group is finished, write your equation on the board. Enter all the equations from the other groups into your calculator and vote on which one is the best fit. Discuss why.

(3) Use the best fit equation determined by the class as the best fit to predict the resistance of a car traveling at 50 mph and 75 mph.

(4) At what speed is the car going when the resistance is 25 lbs? Discuss the method you used to find this.
**Method of Finite Differences**

(1) Evaluate the following table of data using the method of finite differences to determine which data represents a linear, quadratic, or exponential function.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
<th>g(x)</th>
<th>h(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>19</td>
<td>18</td>
</tr>
</tbody>
</table>

Linear: ___________  Quadratic: ___________  Exponential: ___________

(2) Discuss what happens in the method of finite differences for an exponential function.

(3) Explain the limitations of predictions based on organized sample sets of data. (i.e. Why can’t you use Method of Finite Differences on real world data?)

(4) Make a scatter plot on your calculator and find the regression equations for each by either changing constants in \[ Y = \] or using the Transformation APPS. (*Hint: The linear function is in the form \( y = mx + b \), the quadratic function is in the form \( y = x^2 + b \), and the exponential function is in the form \( f(x) = b^x + D \).)

\[ f(x) = \] ___________  \[ g(x) = \] ___________  \[ h(x) = \] ___________

(5) Use the regression feature of your calculator to find the exponential regression equation (\[ \text{STAT, CALC, 0: ExpReg L1, L2, Y1} \]) and discuss the differences. Which is better, yours or the calculators?
Real World Exponential Data

Enter the following data into your calculator:

Wind tunnel experiments are used to test the wind friction or resistance of an automobile at the following speeds.

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>Resistance (lbs)</th>
</tr>
</thead>
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<td>17.0</td>
</tr>
<tr>
<td>40</td>
<td>22.4</td>
</tr>
<tr>
<td>45</td>
<td>30.2</td>
</tr>
<tr>
<td>55</td>
<td>59.2</td>
</tr>
</tbody>
</table>

Determine a regression equation for the data changing the constants $A$ and $B$ in $f(x) = AB^x$ to find the best equation to fit the data. Change the individual constants until your graph matches the data or use the Transformation APPS. Do not use the regression feature of the calculator. Then enter your equation to see if it matches the data.

(1) Write your equation and discuss why you chose the values $A$ and $B$.

*Answers will vary* $f(x) = 3.493(1.050)^x$

(2) When each group is finished, write your equation on the board. Enter all the equations from the other groups into your calculator and vote on which one is the best fit. Discuss why.

*Discussions will vary.*

(3) Use the best fit equation determined by the class as the best fit to predict the resistance of a car traveling at 50 mph and 75 mph.

$f(50) = 40.058$ lbs, $f(75) = 135.660$ lbs.

(4) At what speed is the car going when the resistance is 25 lbs? Discuss the method you used to find this.

The car will be going approximately 40.338 mph. Some students will trace to a $y = 25$, but the most accurate way is to graph the line $y = 25$ and find the point of intersection.
Unit 6, Activity 3, Exponential Regression Equations with Answers

Method of Finite Differences

(1) Evaluate the following table of data using the method of finite differences to determine which data represents a linear, quadratic, or exponential function.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
<th>g(x)</th>
<th>h(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>19</td>
<td>18</td>
</tr>
</tbody>
</table>

(2) Discuss what happens in the method of finite differences for an exponential function.

In an exponential function, the set differences are always the same each time you subtract.

(3) Explain the limitations of predictions based on organized sample sets of data. (i.e. Why can’t you use Method of Finite Differences on real-world data?)

Real world data is not exactly exponential therefore the differences will vary.

(4) Make a scatter plot on your calculator and find the regression equations for each by either changing constants in $y =$ or using the Transformation APPS. (Hint: The linear function is in the form $y = mx + b$, the quadratic function is in the form $y = x^2 + b$, and the exponential function is in the form $f(x) = bx + D$.)

$f(x) = 2x + 3 \quad g(x) = x^2 + 3 \quad h(x) = 2^x + 2$

(5) Use the regression feature of your calculator to find the exponential regression equation ($\text{STAT}, \text{CALC}, 0: \text{ExpReg}$ L1, L2, Y1) and discuss the differences. Which is better, yours or the calculators?

$y = 2.702(1.568)^x$. This equation did not have a vertical shift but changes the leading coefficient. My equation $y = 2^x + 2$ matched the data better.
Unit 6, Activity 4, Exponential Data Research Project

Name__________________________________________________ Due Date________________________

When It Grows, It Grows Fast

This is an individual project, and each student must have different data, so be the first to print out your data and claim the topic. Make sure the data creates an exponential scatter plot.

Possible topics include: US Bureau of Statistics, Census, Stocks, Disease, Bacteria Growth, Investments, Land Value, Animal Population, number of stamps produced each year.

Directions:
(1) Search the Internet or newspaper and find data that is exponential in nature. You must have at least ten data points. Print out the data making sure to include the source and date of your data and bring to class to claim your topic.
(2) Plot the data using either your calculator or an Excel spreadsheet.
(3) Print out the table of data and the graph from your calculator or spreadsheet.
(4) Find the mathematical model (regression equation) for the data and state a reasonable domain and range for the topic.
(5) Compose a relevant question that can be answered using your model to extrapolate, to make a future prediction, and to answer the question.
(6) Type a paragraph (minimum five sentences) about the subject of your study. Discuss any limitations on using the data for predictions.
(7) Include all the above information on a ½ sheet of poster paper with an appropriate title, your name, date, and period.
(8) Present findings to the class.

Grading Rubric for Exponential Data Research Project

10 pts. – table of data with proper documentation (source and date of data)
10 pts. – scatterplot with model equation from the calculator or spreadsheet (not by hand)
10 pts. – equations, domain, range
10 pts. – real world problem using extrapolation with correct answer
10 pts. – discussion of subject and limitations of the prediction
10 pts. – poster - neatness, completeness, readability
10 pts. – class presentation
Characteristics of Parent Logarithm Graph

(1) Graph \( f(x) = \log x \) by plotting points by hand on the graph below for \( x = 1, 10, 100, \) and 0.1, and connecting the dots.

(2) Discuss the shape of the graph, its speed of increasing, its domain, range, asymptotes, end-behavior, and intercepts.

(3) Graph \( g(x) = \log_5 x \) by plotting points by hand on the same graph for \( x = 1, 5, 25, \text{ and } \frac{1}{5} \), and connecting the dots. Discuss the similarities and differences in the graphs when changing the base. Why were these \( x \)-values chosen to plot?

(4) Predict what a graph of log base 20 would look like. What \( x \)-values would you choose to plot?
Characteristics of Parent Logarithm Graph

The graphs below contain the graph of \( f(x) = \log x \). Graph the following functions by hand on these graphs using your knowledge of shifts and translations in the form \( f(x) = A \log B(x - C) + D \). Label the asymptotes and \( x \)- and \( y \)-intercepts when possible.

(1) \( f(x) = \log (x - 2) \)

(2) \( f(x) = \log x + 2 \)

(3) \( f(x) = \log 10x \)

(4) \( f(x) = 3 \log x \)

(5) \( f(x) = -\log x \)

(6) \( f(x) = \log(-x) \)

(7) Discuss the similarities and differences in #2 and #3.

(8) Analytically find the \( x \)-intercepts of all the functions. Show your work.

(9) Discuss domain restrictions on #1. Find the domain of \( g(x) = \log(x + 1) \) and discuss why you cannot find \( g(-2) \) but you can find \( g(0) \). Can you find \( f(0) \)?
Characteristics of Parent Logarithm Graph

(1) Graph \( f(x) = \log x \) by plotting points by hand on the graph below for \( x = 1, 10, 100, \) and 0.1, and connecting the dots. **Graph in black with ordered pairs (1, 0), (10, 1), (100, 2), (0.1, -1)**

![Graph of log x](image)

(2) Discuss the shape of the graph, its speed of increasing, its domain, range, asymptotes, end-behavior, and intercepts.

The graph increases very fast from 0 to 1, but then very slowly after 1. The domain is \( x > 0 \), range – all reals, asymptote \( x = 0 \). End-behavior: as \( x \to 0, y \to -\infty \). As \( x \to \infty, y \to \infty \). There is no \( y \)-intercept. The \( x \)-intercept is (1, 0).

(3) Graph \( g(x) = \log_5 x \) by plotting points by hand on the same graph for \( x = 1, 5, 25, \) and \( \frac{1}{5} \), and connecting the dots. Discuss the similarities and differences in the graphs when changing the base. Why were these \( x \)-values chosen to plot?

**Graph in red. Ordered pairs (1, 0), (5, 1), (25, 2), \( \left( \frac{1}{5}, -1 \right) \). Both graphs have an \( x \)-intercept at (1, 0), domain, range, vertical asymptote and end behavior, but \( \log_5 x \) has higher \( y \)-values for every \( x \) value \( \neq 1 \) so there is a vertical stretch. These \( x \) values are powers of 5.**

(4) Predict what a graph of log base 20 would look like. What \( x \)-values would you choose to plot? **It would have the same \( x \)-intercept, domain, range, vertical asymptote, and end-behavior, but the \( y \)-values would be smaller for every \( x \) value \( \neq 1 \) so there would be a vertical shrink. \( x = \frac{1}{20}, 1, 20, 400 \)**
Characteristics of Parent Logarithm Graph

The graphs below contain the graph of \( f(x) = \log x \). Graph the following functions by hand on these graphs using your knowledge of shifts and translations in the form \( f(x) = A \log B(x - C) + D \). Label the asymptotes and \( x \)- and \( y \)-intercepts when possible.

1. \( f(x) = \log(x - 2) \)
2. \( f(x) = \log x + 2 \)
3. \( f(x) = \log 10x \)
4. \( f(x) = 3 \log x \)
5. \( f(x) = -\log x \)
6. \( f(x) = \log(-x) \)

7. Discuss the similarities and differences in \#2 and \#3. They both look like they were shifted up but actually \#2 was shifted up. In \#3 the domain was compressed. The point \((10, 1)\) moved to \((1, 1)\) and \((100, 2)\) moved to \((10, 2)\).

8. Analytically find the \( x \)-intercepts of all the functions. Show your work

\[
\begin{align*}
(1) \quad & 0 = \log(x - 2) \Rightarrow 10^0 = x - 2 \quad \therefore x = 3. \\
(2) \quad & 0 = \log x + 2 \Rightarrow -2 = \log x \quad \therefore 10^{-2} = x \\
(3) \quad & 0 = \log 10x \Rightarrow 10^0 = 10x \quad \therefore x = 1 \\
(4) \quad & 0 = 3 \log x \Rightarrow 0 = \log x \Rightarrow 10^0 = x \quad \therefore x = 1 \\
(5) \quad & 0 = -\log x \Rightarrow 0 = \log x \Rightarrow 10^0 = x \quad \therefore x = 1 \\
(6) \quad & 0 = \log(-x) \Rightarrow 10^0 = -x \quad \therefore x = -1
\end{align*}
\]

9. Discuss domain restrictions on \#1. Find the domain of \( g(x) = \log(x + 1) \) and discuss why you cannot find \( g(-2) \) but can find \( g(0) \). Can you find \( f(0) \)?

The domain of the parent function \( f(x) = \log x \) is \( x > 0 \). Since the graph was shifted to the right 2, the domain was shifted to \( x > 2 \). The domain of \( g(x) \) will be \( x > -1 \). \(-2\) is not in the domain but \( 0 \) is. You cannot find \( f(0) \) because \( 0 \) is not in the domain of the parent function.
**Unit 6, Activity 10, Exponential Growth and Decay Lab**

Name__________________________________________ Date______________________

**Exponential Growth** Get a cup of about 50 Skittles® (or M & M’s®). Start with 6 Skittles®. Pour out the Skittles®. Assume that the ones with the S showing have had babies and add that many more Skittles® to the cup. Repeat the process until all 50 Skittles® have been used.

<table>
<thead>
<tr>
<th>Year</th>
<th>Population (Number you have after adding babies.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
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</tbody>
</table>

(1) Create a scatter plot and find the exponential regression equation on your calculator. (Round numbers 3 places behind the decimal.) ________________________________

(2) Use your regression equation to predict the population in 20 years. _________________

(3) In what year will the population be 100? _________________________________

(4) What is your correlation coefficient? _____________ Is this a good correlation? _________

**Exponential Decay** Pour out the Skittles® and remove the ones with the S showing as these represent an organism that has contacted a radioactive substance and has died. Repeat the process until one candy is left.

<table>
<thead>
<tr>
<th>Year</th>
<th>Population (Number left without S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(1) Create a scatter plot and find the exponential regression equation on your calculator. (Round numbers 3 places behind the decimal.) ________________________________

(2) Use your regression equation to predict the population in 20 years. _________________

(3) In what year will the population be 100? _________________________________

(4) What is your correlation coefficient? _____________ Is this a good correlation? _________
**Unit 6, Activity 13, Money in the Bank Research Project**

Name__________________________________ Due Date________________________

In this research project, you will choose a financial institution in town or on the Internet. Each student in a class must choose a different bank, so claim your bank early. Contact the bank or go online to find out information about the interest rates available for two different types of accounts, and how they are compounded. Fill in the following information and solve the following problems. When all projects are in, you will report to the class.

**Information Sheet:** Name of bank, name of person you spoke to, bank address, and phone number or the URL if online, types of accounts, interest rates, and how funds are compounded.

**Problem:** Create a hypothetical situation in which you invest $500.

1. Find the equation to model two different accounts for your bank.
2. Determine how much you will have at the end of high school, at the end of college, and when you retire after 50 years, for each account. (Assume you finish high school in one year and college four years later.)
3. Determine how many years it will take you to double your money for each account.
4. Determine in which account you will put your money and discuss why.

**Class Presentation:** Display all information on a poster board and report to the class.

**Grading Rubric for Data Research Project:**

10 pts.  – Information sheet: Name of bank, name of person you spoke to, bank address, and phone number or the URL if online, types of accounts, interest rates, and how funds are compounded (source and date of data)

10 pts.  – Compound interest equation for each situation; account value for both accounts at the end of high school, college, and when you retire in 50 years (show all your work)

10 pts.  – Solution showing your work of how long it will take you to double your money in each account

10 pts.  – Discussion of where you will put your money and why

10 pts.  – Poster - neatness, completeness, readability

10 pts.  – Class presentation
7.1 Basic Graphs – graph and locate \( f(I) = y = x, x^2, x^3, \sqrt{x}, \sqrt[3]{x}, |x|, \frac{1}{x}, \lfloor x \rfloor, \log x, 2^x \).

7.2 Continuity – provide an informal definition and give examples of continuous and discontinuous functions.

7.3 Increasing, Decreasing, and Constant Functions – write definitions and draw example graphs such as \( y = \sqrt{9 - x^2} \), state the intervals on which the graphs are increasing and decreasing.

7.4 Even and Odd Functions – write definitions and give examples, illustrate properties of symmetry, and explain how to prove that a function is even or odd (e.g., prove that \( y = x^4 + x^2 + 2 \) is even and \( y = x^3 + x \) is odd).

7.5 General Piecewise Function – write the definition and then graph, find the domain and range, and solve the following ex. \( f(x) = \begin{cases} 2x + 1 & \text{if } x \geq 5 \\ -x^2 & \text{if } x < 5 \end{cases} \) for \( f(4) \) and \( f(1) \).

For properties 7.6 – 7.9 below, do the following:
- Explain in words the effect on the graph.
- Give an example of the graph of a given abstract function, and then the function transformed (do not use \( y = x \) as your example).
- Explain in words the effect on the domain and range of a given function. Use the domain \([-2, 6]\) and the range \([-8, 4]\) to find the new domain and range of the transformed function.

7.6 Translations \( f(x + k) \) and \( f(x - k) \), \( f(x) + k \) and \( f(x) - k \)

7.7 Reflections \( f(-x) \) and \( -f(x) \)

7.8 Dilations \( f(kx) \), \( \sqrt{k} < 1 \) and \( \sqrt{k} > 1 \), \( k f(x) \) \( \sqrt{k} < 1 \) and \( \sqrt{k} > 1 \)

7.9 Reflections \( f(|x|) \) and \( |f(x)| \)
Graph the following by hand and locate the zeroes and $f(1)$.

1. $f(x) = x$
2. $f(x) = x^2$
3. $f(x) = \sqrt{x}$
4. $f(x) = x^3$
5. $f(x) = |x|$
6. $f(x) = 2^x$
7. $f(x) = \frac{1}{x}$
8. $f(x) = \sqrt[3]{x}$
9. $f(x) = \log x$
10. $f(x) = \lfloor x \rfloor$
Unit 7, Activity 2, Translations

Use the abstract graphs of \( g(x) \) below to answer questions #1 – 5.

(1) What is the domain of \( g(x) \)? \(_{\text{domain}}\) range? \(_{\text{range}}\)

Draw the graph of the following over the graph of \( g(x) \), label the new points, and find the new domain and range:

(2) \( g(x) + 3 \)

D: \(_{\text{domain}}\) R: \(_{\text{range}}\)

(3) \( g(x) - 3 \)

D: \(_{\text{domain}}\) R: \(_{\text{range}}\)

(4) \( g(x + 3) \)

D: \(_{\text{domain}}\) R: \(_{\text{range}}\)

(5) \( g(x - 3) \)

D: \(_{\text{domain}}\) R: \(_{\text{range}}\)

Practice without a graph: If the domain of \( f(x) \) is \([-4, 10]\) and the range is \([-6, 5]\) find the domains and ranges of the following. If they do not change, write “same.”

(6) \( f(x - 8) \)

D: \(_{\text{domain}}\) R: \(_{\text{range}}\)

(7) \( f(x) - 8 \)

D: \(_{\text{domain}}\) R: \(_{\text{range}}\)
Parent Function Shifts

State the parent function \( f(x) \) and the domain and range of the parent function. Graph the parent function and the shifted function by hand and state the new domain and range.

(8) \( j(x) = \sqrt{x + 2} - 3 \) parent \( f(x) = \) ______

\[
f(x) \quad \text{D: } \quad \text{R: }
\]

\[
j(x) \quad \text{D: } \quad \text{R: }
\]

(9) \( k(x) = \frac{1}{x - 3} + 2 \) parent \( f(x) = \) ______

\[
f(x) \quad \text{D: } \quad \text{R: }
\]

\[
k(x) \quad \text{D: } \quad \text{R: }
\]

(10) \( h(x) = |x - 5| - 7 \) parent \( f(x) = \) ______

\[
f(x) \quad \text{D: } \quad \text{R: }
\]

\[
h(x) \quad \text{D: } \quad \text{R: }
\]

(11) \( t(x) = \log(x + 4) + 3 \) parent \( f(x) = \) ______

\[
f(x) \quad \text{D: } \quad \text{R: }
\]

\[
t(x) \quad \text{D: } \quad \text{R: }
\]
**Unit 7, Activity 2, Translations with Answers**

Name ___________________  Key ___________________  Date ___________________

**Abstract Shifts**

Use the abstract graphs of $g(x)$ below to answer questions #1 – 5.

(1) What is the domain of $g(x)$? ______ $[-5, 4]$ ______ range? ______ $[-3, 8]$ ______

Draw the graph of the following over the graph of $g(x)$, label the new points, and find the new domain and range:

(2) $g(x) + 3$  D: ______ $[-5, 4]$  R: ______ $[0, 11]$ ______

(3) $g(x) - 3$  D: ______ $[-5, 4]$  R: ______ $[-6, 5]$ ______

(4) $g(x + 3)$  D: ______ $[-8, 1]$  R: ______ $[-3, 8]$ ______

(5) $g(x - 3)$  D: ______ $[6, 8]$  R: ______ $[-3, 8]$ ______

Practice without a graph: If the domain of $f(x)$ is $[-4, 10]$ and the range is $[-6, 5]$ find the domains and ranges of the following. If they do not change, write “same”:

(6) $f(x - 8)$  D: ______ $[4, 18]$  R: ______ same ______

(7) $f(x) - 8$  D: ______ same ______  R: ______ $[-14, -3]$ ______
Parent Function Shifts

State the parent function \( f(x) \) and the domain and range of the parent function. Graph the parent function and the shifted function by hand and state the new domain and range.

(8) \( j(x) = \sqrt{x + 2} - 3 \) parent \( f(x) = \sqrt{x} \)

\[
\begin{align*}
  f(x) & : [0, \infty) & R : [0, \infty) \\
  j(x) & : [-2, \infty) & R : [-3, \infty)
\end{align*}
\]

(9) \( k(x) = \frac{1}{x - 3} + 2 \) parent \( f(x) = \frac{1}{x} \)

\[
\begin{align*}
  f(x) & : x \neq 0 & R : y \neq 0 \\
  k(x) & : x \neq 3 & R : y \neq 2
\end{align*}
\]

(10) \( h(x) = |x - 5| - 7 \) parent \( f(x) = \)_____

\[
\begin{align*}
  f(x) & : all \, reals & R : [0, \infty) \\
  h(x) & : all \, reals & R : [-7, \infty)
\end{align*}
\]

(11) \( t(x) = \log(x + 4) + 3 \) parent \( f(x) = \)_____

\[
\begin{align*}
  f(x) & : (0, \infty) & R : all \, reals \\
  t(x) & : (-4, \infty) & R : all \, reals
\end{align*}
\]
Unit 7, Activity 3, Reflections Discovery Worksheet

Name__________________________ Date_________________

**Reflections**

Graph the functions from the Bellringer with your graphing calculator and sketch below:

1. \( f(x) = \sqrt{x} \)  
2. \( g(x) = -\sqrt{x} \)  
3. \( h(x) = \sqrt{-x} \)

(4) What is the effect of \(-f(x)\)?

Sketch the following without a calculator:

5. \( f(x) = -x^2 \)  
6. \( f(x) = -\left(\sqrt{x + 3}\right) \)

Graph the following functions with your graphing calculator and sketch below:

7. \( f(x) = 2^x \)  
8. \( g(x) = 2^{-x} \)

(9) Compare #1 with #3 and #7 with #8. What is the effect of \(f(-x)\)?

Sketch the following without a calculator:

10. \( f(x) = (-x)^2 \)  
11. \( f(x) = \sqrt{-x} + 3 \)

If the function \( h(x) \) has a domain \([-4, 6]\) and range \([-3, 10]\), what is the domain and range of

12. \(-h(x)\)? D:___________ R:___________  
13. \(-f(x)\)? D:___________ R:___________
**Unit 7, Activity 3, Reflections Discovery Worksheet with Answers**

**Reflections**

Graph the functions from the Bellringer with your graphing calculator and sketch below:

(1) \( f(x) = \sqrt{x} \)  

(2) \( g(x) = -\sqrt{x} \)  

(3) \( h(x) = \sqrt{-x} \)  

(4) What is the effect of \(-f(x)\)? **Reflects the graph across the x-axis**

Sketch the following without a calculator:

(5) \( f(x) = -x^2 \)  

(6) \( f(x) = -\left(\sqrt{x} + 3\right) \)  

Graph the following functions with your graphing calculator and sketch below:

(7) \( f(x) = 2^x \)  

(8) \( g(x) = 2^{-x} \)  

(9) Compare #1 with #3 and #7 with #8. What is the effect of \( f(-x) \)? **Reflects graph across y-axis**

Sketch the following without a calculator:

(10) \( f(x) = (-x)^2 \)  

(11) \( f(x) = \sqrt{-x} + 3 \)  

If the function \( h(x) \) has a domain \([-4, 6]\) and range \([-3, 10]\), what is the domain and range of

(12) \(-h(x)\)? D: **same**  

R: \([-10, 3]\)  

(13) \( h(-x) \)? D: \([-6, 4]\)  

R: **same**
Unit 7, Activity 3, Dilations Discovery Worksheet

Name__________________________________________ Date____________________

Dilations

Graph the following functions with your graphing calculator and sketch below:

(14) \( f(x) = \sqrt{9-x^2} \)  

(15) \( g(x) = 3\sqrt{9-x^2} \)  

(16) \( h(x) = \frac{1}{3}\sqrt{9-x^2} \)

(17) \( s(x) = \sqrt{9-(3x)^2} \)  

(18) \( j(x) = \sqrt{9-\left(\frac{1}{3}x\right)^2} \)

(19) What is the effect of \( kf(x) \) if \( k > 1 \)?__________________________

if \( 0 < k < 1 \)?__________________________

(20) What is the effect of \( f(kx) \) if \( k > 1 \)?__________________________

if \( 0 < k < 1 \)?__________________________

(21) Which one affects the domain?__________________________ range?__________________________

Sketch the following without your calculator for \(-6 < x < 6 \) and \(-4 < y < 4 \) and find \((1\frac{1}{2}, f(1\frac{1}{2}))\):

(22) \( t(x) = \left\lfloor x \right\rfloor \)  

(23) \( f(x) = \left\lfloor 2x \right\rfloor \)  

(24) \( g(x) = 2\left\lfloor x \right\rfloor \)

If the function \( h(x) \) has a domain \([-4, 6]\) and range \([-3, 10]\), what is the domain and range of

(25) \( 5h(x) \)? D:___________ R:_________  

(26) \( \frac{1}{5}h(x) \)? D:___________ R:_________

(27) \( h(5x) \)? D:___________ R:_________  

(28) \( h\left(\frac{1}{5}x\right) \)? D:___________ R:_________
**Dilations**

Graph the following functions with your graphing calculator and sketch below:

14. \( f(x) = \sqrt{9 - x^2} \)

15. \( g(x) = 3\sqrt{9 - x^2} \)

16. \( h(x) = \frac{1}{3}\sqrt{9 - x^2} \)

17. \( s(x) = \sqrt{9 - (3x)^2} \)

18. \( j(x) = \sqrt{9 - \left(\frac{1}{3}x\right)^2} \)

Graph #17 does touch the \(x\)-axis, but it looks like it does not because there are no pixels near the zeroes.

19. What is the effect of \( k \cdot f(x) \) if \( k > 1 \)? **stretches the graph vertically**

if \( 0 < k < 1 \)? **compresses the graph vertically**

20. What is the effect of \( f(kx) \) if \( k > 1 \)? **compresses the graph horizontally**

if \( 0 < k < 1 \)? **stretches the graph horizontally**

21. Which one affects the domain? \( f(kx) \) range? \( k \cdot f(x) \)

Sketch the following without your calculator for \(-6 < x < 6\) and \(-4 < y < 4\) and find \((1\frac{1}{2}, f(1\frac{1}{2}))\):

22. \( t(x) = \lfloor x \rfloor \)

23. \( f(x) = \lfloor 2x \rfloor \)

24. \( g(x) = 2 \lfloor x \rfloor \)

If the function \( h(x) \) has a domain \([-4, 6]\) and range \([-3, 10]\), what is the domain and range of

25. \( 5h(x) \)? D: **same** R: \([\frac{-15}{5}, 50]\)

26. \( \frac{1}{5}h(x) \)? D: **same** R: \(\left[-\frac{3}{5}, 2\right]\)

27. \( h(5x) \)? D: \(\left[-\frac{4}{5}, \frac{6}{5}\right]\) R: **same**

28. \( h\left(\frac{1}{5}x\right) \)? D: \([\frac{-20}{5}, 30]\) R: **same**
Abstract Reflections & Dilations

Domain of \( g(x) \): __________ Range of \( g(x) \): __________

Draw the graph of the following over the graph of \( g(x) \), label the new points, and find the new domain and range:

(29) \( g(-x) \)  
\[ D: \quad R: \]

(30) \( -g(x) \)  
\[ D: \quad R: \]

(31) \( 2g(x) \)  
\[ D: \quad R: \]

(32) \( \frac{1}{2} g(x) \)  
\[ D: \quad R: \]

(33) \( g(2x) \)  
\[ D: \quad R: \]

(34) \( g\left(\frac{1}{2}x\right) \)  
\[ D: \quad R: \]
**Unit 7, Activity 3, Abstract Reflections & Dilations with Answers**

Name_________________________________________ Date____________________

**Abstract Reflections and Dilations**

Domain of \(g(x)\): \([-5, 4]\)  Range of \(g(x)\): \([-3, 8]\)  Draw the graph of the following over the graph of \(g(x)\), label the new points, and find the new domain and range:

(29) \(g(-x)\)  D: \([-4, 5]\)  R: same

(30) \(-g(x)\)  D: same  R: \([-8, 3]\)

(31) \(2g(x)\)  D: same  R: \([-6, 16]\)

(32) \(\frac{1}{2}g(x)\)  D: same  R: \([-1.5, 4]\)

(33) \(g(2x)\)  D: \([-2.5, 2]\)  R: same

(34) \(g(\frac{1}{2}x)\)  D: \([-10, 8]\)  R: same

*Blackline Masters, Algebra II
Louisiana Comprehensive Curriculum, Revised 2008*
Unit 7, Activity 5, Tying It All Together

Name_________________________________________ Date_____________________________________

Reflections, Dilations, and Translations

\[ f(x) = \]

I. Graphing: Given the graph of the function \( f(x) \), match the following shifts and translations.

1. (1) \( 2f(x) \)
2. (2) \( f(2x) \)
3. (3) \( -f(x) \)
4. (4) \( f(-x) \)
5. (5) \( |f(x)| \)
6. (6) \( f(|x|) \)
7. (7) \( f(x) + 4 \)
8. (8) \( f(x + 4) \)

II. Domains and Ranges: Write the new domain and range if \( g(x) \) has a domain of \([-10, 4]\) and the range is \([-6, 8]\). If there is no change, write “same.” If it cannot be determined, write “CBD.”

<table>
<thead>
<tr>
<th>DOMAIN:</th>
<th>RANGE:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a) ( g(x) + 1 )</td>
<td>_______ _______</td>
</tr>
<tr>
<td>b) ( g(x) - 4 )</td>
<td>_______ _______</td>
</tr>
<tr>
<td>2a) ( g(x + 1) )</td>
<td>_______ _______</td>
</tr>
<tr>
<td>b) ( g(x - 4) )</td>
<td>_______ _______</td>
</tr>
<tr>
<td>3a) ( g(2x) )</td>
<td>_______ _______</td>
</tr>
<tr>
<td>b) ( 2g(x) )</td>
<td>_______ _______</td>
</tr>
<tr>
<td>4a) ( g(\frac{x}{2}) )</td>
<td>_______ _______</td>
</tr>
<tr>
<td>b) ( \frac{1}{2}g(x) )</td>
<td>_______ _______</td>
</tr>
<tr>
<td>5a) ( -g(x) )</td>
<td>_______ _______</td>
</tr>
<tr>
<td>b) ( g(-x) )</td>
<td>_______ _______</td>
</tr>
<tr>
<td>6a) (</td>
<td>g(x)</td>
</tr>
<tr>
<td>b) ( g(</td>
<td>x</td>
</tr>
</tbody>
</table>
**Unit 7, Activity 5, Tying It All Together with Answers**

Name ___________________________     Date ________________________

### Reflections, Dilations, and Translations

**Graph:**

$f(x) =$

**I. Graphing:** Given the graph of the function $f(x)$, match the following shifts and translations.

- **C** (1) $2f(x)$
- **H** (2) $f(2x)$
- **D** (3) $-f(x)$
- **E** (4) $f(-x)$
- **A** (5) $|f(x)|$
- **B** (6) $f(|x|)$
- **J** (7) $f(x) + 4$
- **L** (8) $f(x + 4)$

**II. Domains and Ranges:** Write the new domain and range if $g(x)$ has a domain of $[-10, 4]$ and the range is $[-6, 8]$. If there is no change, write “same.” If it cannot be determined, write “CBD.”

<table>
<thead>
<tr>
<th><strong>1a)</strong></th>
<th><strong>1b)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(x) + 1$</td>
<td>same</td>
</tr>
<tr>
<td>$g(x) - 4$</td>
<td>same</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>2a)</strong></th>
<th><strong>2b)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(x + 1)$</td>
<td>$[-11, 3]$</td>
</tr>
<tr>
<td>$g(x - 4)$</td>
<td>$[-6, 8]$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>3a)</strong></th>
<th><strong>3b)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(2x)$</td>
<td>$[-5, 2]$</td>
</tr>
<tr>
<td>$2g(x)$</td>
<td>same</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>4a)</strong></th>
<th><strong>4b)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(\frac{1}{2}x)$</td>
<td>$[-20, 8]$</td>
</tr>
<tr>
<td>$\frac{1}{2}g(x)$</td>
<td>same</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>5a)</strong></th>
<th><strong>5b)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$-g(x)$</td>
<td>same</td>
</tr>
<tr>
<td>$g(-x)$</td>
<td>$[-4, 10]$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>6a)</strong></th>
<th><strong>6b)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>g(x)</td>
</tr>
<tr>
<td>$g(</td>
<td>x</td>
</tr>
</tbody>
</table>
Graphing Piecewise Functions

In #1 – 4, graph and state the domain and range, \( x \)- and \( y \)-intercepts, the intervals on which the function is increasing, decreasing, or constant, and if the function is continuous:

1. \( f(x) = \begin{cases} \sqrt{x-4} & \text{if } x \geq 4 \\ x^2 - 8 & \text{if } x < 4 \end{cases} \)

2. \( f(x) = \begin{cases} 2^{x+1} & \text{if } x \geq -1 \\ -3|x+4| + 6 & \text{if } x < -1 \end{cases} \)

3. \( f(x) = \begin{cases} x^3 - 5 & \text{if } x < 0 \\ \frac{1}{x} - 1 & \text{if } 0 \leq x \leq 4 \end{cases} \)

4. \( f(x) = \begin{cases} (x-3)^2 - 2 & \text{if } 0 \leq x \leq 5 \\ \log(x+10) & \text{if } x < 0 \end{cases} \)

5. Graph \( h(x) \) and find the \( a \) and \( b \) that makes the function continuous: \( a = \quad b = \)

\[ h(x) = \begin{cases} \frac{1}{x-1} & x > 2 \\ ax + b & 0 \leq x \leq 2 \\ -\sqrt{x} & x < 0 \end{cases} \]
Analyzing Graphs of Piecewise Functions

(6) Write a piecewise function for the graph of \( g(x) \) below. (Assume all left endpoints are included and all right endpoints are not included.)

\[
\begin{align*}
g(x) &= \begin{cases} 
\text{ } & \text{ if } \text{ } \\
\text{ } & \text{ if } \text{ } \\
\text{ } & \text{ if } \text{ }
\end{cases}
\]

(7) Using the graph of \( g(x) \) above, draw the graph \( h(x) = g(x+4) - 5 \) and write its piecewise function.

\[
\begin{align*}
h(x) &= \begin{cases} 
\text{ } & \text{ if } \text{ } \\
\text{ } & \text{ if } \text{ } \\
\text{ } & \text{ if } \text{ }
\end{cases}
\]

(8) Using the graph of \( g(x) \) to the right, draw the graph of \( t(x) = \frac{1}{2}g(4x) \) and write its piecewise function.

\[
\begin{align*}
t(x) &= \begin{cases} 
\text{ } & \text{ if } \text{ } \\
\text{ } & \text{ if } \text{ } \\
\text{ } & \text{ if } \text{ }
\end{cases}
\]

(9) Brett is on the ground outside the stadium and throws a baseball to John at the top of the stadium 36 feet above the ground. Brett throws with an initial velocity of 60 feet/sec. It goes above John’s head, and he catches it on the way down. John holds the ball for 5 seconds then drops it to Brett. Graph the function and find a piecewise function that models the height of the ball \( s(t) \) over time \( t \) in seconds after Brett throws the ball. (Remember the quadratic equation from Unit 5 for position of a free falling object if acceleration due to gravity is \(-32 \text{ ft/sec}^2\): \( s(t) = -16t^2 + v_0t + s_0 \).)

(a) How long after Brett threw the ball did John catch it?

(b) How high did the ball go?

(c) At what time did the ball hit the ground?

\[
\begin{align*}
s(t) &= \begin{cases} 
\text{ } & \text{ if } \text{ } \\
\text{ } & \text{ if } \text{ } \\
\text{ } & \text{ if } \text{ }
\end{cases}
\]
Graphing Piecewise Functions

In #1 – 4, graph and state the domain and range, x- and y-intercepts, the intervals on which the function is increasing, decreasing, or constant, and if the function is continuous:

(1) \[ f(x) = \begin{cases} \sqrt{x-4} & \text{if } x \geq 4 \\ x^2 - 8 & \text{if } x < 4 \end{cases} \]

(2) \[ f(x) = \begin{cases} 2^{x+1} & \text{if } x \geq -1 \\ -3|x+4|+6 & \text{if } x < -1 \end{cases} \]

(3) \[ f(x) = \begin{cases} x^3 - 5 & \text{if } x < 0 \\ \frac{1}{3}x + 1 & \text{if } 0 \leq x \leq 4 \\ -1 & \text{if } x > 4 \end{cases} \]

(4) \[ f(x) = \begin{cases} |x-3|^2 - 2 & \text{if } 0 < x \leq 5 \\ \log(x+10) & \text{if } x \leq 0 \end{cases} \]

(5) Graph \[ h(x) \] and find the \( a \) and \( b \) that makes the function continuous: \( a = \frac{1}{2}, \ b = 0 \)

\[ h(x) = \begin{cases} \frac{1}{x-1} & x > 2 \\ ax + b & 0 \leq x \leq 2 \\ -\sqrt{x} & x < 0 \end{cases} \]
Analyzing Graphs of Piecewise Functions

(6) Write a piecewise function for the graph of \( g(x) \) below. (Assume all left endpoints are included and all right endpoints are not included.)

\[
g(x) =\begin{cases} 
  x + 8 & -6 \leq x < 0 \\
  (x - 2)^2 & 0 \leq x < 4 \\
  6.5 & 4 \leq x < 9
\end{cases}
\]

(7) Using the graph of \( g(x) \) above, draw the graph \( h(x) = g(x+4) - 5 \) and write its piecewise function.

\[
h(x) = \begin{cases} 
  x + 7 & -10 \leq x < -4 \\
  (x + 2)^2 - 5 & -4 \leq x < 0 \\
  1.5 & 0 \leq x < 5
\end{cases}
\]

(8) Using the graph of \( g(x) \) to the right, draw the graph of \( t(x) = \frac{1}{2}g(4x) \) and write its piecewise function.

\[
t(x) = \begin{cases} 
  2x + 4 & -1.5 \leq x < 0 \\
  \frac{1}{2}(4x - 2)^2 & 0 \leq x < 1 \\
  3.25 & 1 \leq x < 9 / 4
\end{cases}
\]

(9) Brett is on the ground outside the stadium and throws a baseball to John at the top of the stadium 36 feet above the ground. Brett throws with an initial velocity of 60 feet/sec. It goes above John’s head, and he catches it on the way down. John holds the ball for 5 seconds then drops it to Brett. Graph the function and find a piecewise function that models the height of the ball \( s(t) \) over time \( t \) in seconds after Brett throws the ball. (Remember the quadratic equation from Unit 5 for position of a free falling object if acceleration due to gravity is \(-32 \text{ ft/sec}^2\): \( s(t) = -16t^2 + v_0t + s_o \).)

(a) How long after Brett threw the ball did John catch it? \textbf{3 seconds}

(b) How high did the ball go? \textbf{56.250 feet}

(c) At what time did the ball hit the ground? \textbf{9.500 sec}

\[
s(t) = \begin{cases} 
  -16t^2 + 60t & 0 \leq x < 3 \\
  36 & 3 \leq x < 8 \\
  -16(t - 8)^2 + 36 & 8 \leq x < 9.5
\end{cases}
\]
Reflections Revisited

Graph the following in your notebook without a calculator:

<table>
<thead>
<tr>
<th></th>
<th>( f(x) )</th>
<th>( f(-x) )</th>
<th>( -f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( y =</td>
<td>x</td>
<td>)</td>
</tr>
<tr>
<td>2</td>
<td>( y = x^2 )</td>
<td>( y = (-x)^2 )</td>
<td>( y = -(x^2) )</td>
</tr>
<tr>
<td>3</td>
<td>( y = 2^x )</td>
<td>( y = 2^{-x} )</td>
<td>( y = -(2^x) )</td>
</tr>
<tr>
<td>4</td>
<td>( y = \sqrt{x} )</td>
<td>( y = \sqrt{-x} )</td>
<td>( y = -\sqrt{x} )</td>
</tr>
<tr>
<td>5</td>
<td>( y = [x] )</td>
<td>( y = [-x] )</td>
<td>( y = -[x] )</td>
</tr>
<tr>
<td>6</td>
<td>( y = \frac{1}{x} )</td>
<td>( y = \frac{1}{(-x)} )</td>
<td>( y = -\left(\frac{1}{x}\right) )</td>
</tr>
<tr>
<td>7</td>
<td>( y = x )</td>
<td>( y = (-x) )</td>
<td>( y = -(x) )</td>
</tr>
</tbody>
</table>

Even & Odd Functions Graphically

**Even Function** \( \equiv \) any function in which \( f(-x) = f(x) \)

1. Look at the graphs of the functions above and in your bellringer, then list the parent functions in which the graph of \( f(-x) \) is the same as the graph of \( f(x) \) and are therefore even functions.

2. Looking at the graphs of these even functions, they are symmetric to ________________.

**Odd Function** \( \equiv \) any function in which \( f(-x) = -f(x) \)

1. Look at the graphs of the functions above and in your bellringer, then list the parent functions in which the graph of \( f(-x) \) is the same as the graph of \(-f(x)\) and are therefore odd functions.

2. Looking at the graphs of these odd functions, they are symmetric to ________________.

3. Graph \( g(x) = x^3 + 1 \) without a calculator and determine if it is even or odd, then explain your answer.

4. Which of the parent functions are neither even nor odd?
**Unit 7, Activity 7, Even & Odd Functions Discovery Worksheet**

**Even & Odd Functions Numerically**

Consider the following table of values and determine which functions may be even, odd, or neither.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$g(x)$</th>
<th>$h(x)$</th>
<th>$s(x)$</th>
<th>$t(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3$</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>$-4$</td>
</tr>
<tr>
<td>$-2$</td>
<td>3</td>
<td>$-4$</td>
<td>5</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>$-1$</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>$-4$</td>
<td>$-2$</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>undefined</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>$-5$</td>
<td>2</td>
<td>4</td>
<td>$-2$</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>$-5$</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>$-6$</td>
<td>6</td>
<td>$-6$</td>
<td>$-4$</td>
</tr>
</tbody>
</table>

Even: ________________  Odd: ________________  Neither: ________________

**Even & Odd Functions Analytically**

Seven sets of ordered pairs are not sufficient to prove a function is even or odd. For example, in $h(x)$, $h(-3) = h(3)$, but the rest did not work. In order to prove whether a function is even or odd, substitute $(-x)$ for every $x$ and determine if $f(-x) = f(x)$ or if $f(-x) = -f(x)$ or neither. Analytically determine if the following functions are even or odd, then graph on your calculator to check the symmetry:

1. $f(x) = x^4 - 3x^2 + 5$
2. $f(x) = 4x^3 - x$
3. $f(x) = |x| + 5$
4. $f(x) = |x^3|$
5. $f(x) = \sqrt{|x|}$
6. $f(x) = \log |x|$
7. $f(x) = 3^{|x + 1|}$
8. $f(x) = x^3 - 4x^2$
Reflections Revisited

Graph the following in your notebook without a calculator:

<table>
<thead>
<tr>
<th></th>
<th>f(x)</th>
<th>f(–x)</th>
<th>–f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>y =</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>y = x²</td>
<td>y = (–x)²</td>
<td>y = –(x²)</td>
</tr>
<tr>
<td>3</td>
<td>y = 2x</td>
<td>y = 2x⁻¹</td>
<td>y = –(2x)</td>
</tr>
<tr>
<td>4</td>
<td>y = √x</td>
<td>y = √–x</td>
<td>y = –√x</td>
</tr>
<tr>
<td>5</td>
<td>y = [x]</td>
<td>y = [–x]</td>
<td>y = –[x]</td>
</tr>
<tr>
<td>6</td>
<td>y = 1/x</td>
<td>y = 1/(–x)</td>
<td>y = –(1/x)</td>
</tr>
<tr>
<td>7</td>
<td>y = = x</td>
<td>y = (–x)</td>
<td>y = –(x)</td>
</tr>
</tbody>
</table>

Even & Odd Functions Graphically

**Even Function** ≡ any function in which \( f(–x) = f(x) \)

1. Look at the graphs of the functions above and in your bellringer, then list the parent functions in which the graph of \( f(–x) \) is the same as the graph of \( f(x) \) and are therefore even functions.

   \[ f(x) = x^2, \quad f(x) = |x| \]

2. Looking at the graphs of these even functions, they are symmetric to **y-axis**.

**Odd Function** ≡ any function in which \( f(–x) = –f(x) \)

1. Look at the graphs of the functions above and in your bellringer, then list the parent functions in which the graph of \( f(–x) \) is the same as the graph of \( –f(x) \) and are therefore odd functions.

   \[ f(x) = x^3, \quad f(x) = \sqrt[3]{x}, \quad f(x) = \frac{1}{x}, \quad f(x) = x \]

2. Looking at the graphs of these odd functions, they are symmetric to **the origin**.

3. Graph \( g(x) = x^3 + 1 \) without a calculator and determine if it is even or odd, then explain your answer.

   **Neither even nor odd. Not symmetric to y-axis nor origin.**

4. Which of the parent functions are neither even nor odd?

   \[ f(x) = \log x, \quad f(x) = 2^x, \quad f(x) = \lfloor x \rfloor \]
**Unit 7, Activity 7, Even & Odd Functions Discovery Worksheet with Answers**

**Even & Odd Functions Numerically**

Consider the following table of values and determine which functions may be even, odd, or neither.

<table>
<thead>
<tr>
<th></th>
<th>f(x)</th>
<th>g(x)</th>
<th>h(x)</th>
<th>s(x)</th>
<th>t(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>–3</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>–4</td>
</tr>
<tr>
<td>–2</td>
<td>3</td>
<td>–4</td>
<td>5</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>–1</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>–4</td>
<td>–2</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>undefined</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>–5</td>
<td>2</td>
<td>4</td>
<td>–2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>–5</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>–6</td>
<td>6</td>
<td>–6</td>
<td>–4</td>
</tr>
</tbody>
</table>

Even: \( f(x) \) and \( t(x) \)  
Odd: \( g(x) \) and \( s(x) \)  
Neither: \( h(x) \)

**Even & Odd Functions Analytically**

Seven sets of ordered pairs are not sufficient to prove a function is even or odd. For example, in \( h(x), \ h(-3) = h(3) \), but the rest did not work. In order to prove whether a function is even or odd, substitute \( -x \) for every \( x \) and determine if \( f(-x) = f(x) \) or if \( f(-x) = -f(x) \) or neither. Analytically determine if the following functions are even or odd, then graph on your calculator to check the symmetry:

1. \( f(x) = x^4 - 3x^2 + 5 \)
   
   \[ f(-x) = (-x)^4 - 3(-x)^2 + 5 = x^4 - 3x^2 + 5 = f(x) \]
   
   \( \therefore \) even, symmetric to \( y \)-axis

2. \( f(x) = x^3 - 4x \)
   
   \[ f(-x) = (-x)^3 + 4(-x) = -x^3 + 4x = -f(x) \]
   
   \( \therefore \) odd, symmetric to the origin

3. \( f(x) = |x| + 5 \)
   
   \[ f(-x) = |-x| + 5 = |x| + 5 = f(x) \]
   
   \( \therefore \) even, symmetric to \( y \)-axis

4. \( f(x) = |x^3| \)
   
   \[ f(-x) = |(-x)^3| = |x^3| = f(x) \]
   
   \( \therefore \) even, symmetric to \( y \)-axis

5. \( f(x) = \sqrt{|x|} \)
   
   \[ f(-x) = \sqrt{|-x|} = \sqrt{|x|} = f(x) \]
   
   \( \therefore \) even, symmetric to \( y \)-axis

6. \( f(x) = \log |x| \)
   
   \[ f(-x) = \log |-x| = \log |x| = f(x) \]
   
   \( \therefore \) even, symmetric to \( y \)-axis

7. \( f(x) = 3^{x+1} \)
   
   \[ f(-x) = 3^{|-x+1|} = 3^{|x-1|} = 3^{x-1} \neq f(-x) \text{ and } f(x) \]
   
   \( \therefore \) neither even nor odd

8. \( f(x) = x^3 - 4x^2 \)
   
   \[ f(-x) = (-x)^3 - 4(-x)^2 = -x^3 - 4x^2 \neq f(-x) \text{ and } f(x) \]
   
   \( \therefore \) neither even nor odd
Data Analysis Research Project

Objectives:
1. Collect data for the past twenty years concerning a topic selected from the list below.
2. Create a mathematical model.
3. Trace the history of the statistics.
4. Evaluate the future impact.
5. Create a *PowerPoint®* presentation of the data including pictures, history, economic impact, spreadsheet data, regression graph and equation, and future predictions.

Possible Topics:
1. # of deaths by carbon monoxide poisoning (overall, in the house, in a car, in a boat)
2. boating accidents or deaths (in LA or in US)
3. jet ski accidents or deaths (in LA or in US)
4. drownings (in LA or in US)
5. number of registered boats (in LA or in US)
6. drunken driving (accidents or deaths, in this parish, LA or in US)
7. DWIs (in this parish, LA or in US)
8. car accidents or deaths (in this town, this parish, LA or in US)
9. suicides (in this town, this parish, LA or in US)
10. census statistics such as population, population by race, marriages, divorces, births, deaths, lifespan, (in this town, this parish, LA or in US)
11. electricity usage (in this town, this parish, LA or in US)
12. land value (in this town, this parish, LA or in US)
13. animal population (in this town, this parish, LA or in US)
14. deaths by any other cause (choose the disease or cause, in this parish, LA or in US)
15. obesity
16. drop out rates
17. building permits for new houses (choose in this town, this parish, LA or in US)

Research:
1. This is an individual or pair project, and each person/pair must have different data.
2. Research on the Internet or other resource to find at least twenty data points; the more data you have, the better the mathematical model will be. The youngest data should be no more than five years ago.
3. Research the historical significance of the data and determine why it would be increasing and decreasing at different times, etc. Determine what might have been happening historically in a year when there is an obvious outlier.
4. Take pictures with a digital camera or find pictures on the Internet to use on your *PowerPoint®*

Calculator/Computer Data Analysis:
1. Enter the data into a spreadsheet or your calculator. Time should be your independent variable using 1 for 1991, 2 for 1992, etc.
2. Create a scatter point chart, find the mathematical model for the data (regression equation or trendline), and find the correlation coefficient (R−squared value on spreadsheet).
**Unit 7, Activity 8, Modeling to Predict the Future**

3. Use a model that has the characteristics you want such as increasing or decreasing, correct end–behavior, zeroes, etc. (For a better regression equation, you may have to eliminate outliers, create two regression equations, one with and one without the outlier, and compare, or create a piecewise function.)

**Extrapolation:**
1. Use your mathematical model to predict what will happen if the trend continues for the next five years and explain the feasibility and limitations of the predictions.
2. Discuss what outliers may occur that would affect this extrapolation.

**Presentation:**
1. Create a six slide PowerPoint® presentation. Make sure to use colors that show well when projected on the screen, and use a large font size.
   Slide 1: Introduction of the topic with a relative picture (not clip art), your name, date, class, hour.
   Slide 2: Statement of the problem, history, and economics of the topic (Use bullets, not sentences, to help you in your oral report – no more than 15 words, bullets should enter PowerPoint® one at a time as you talk.)
   Slide 3: Scatter plot graph of the data-clearly labeled, curve, regression equation, and correlation coefficient. Type regression equation with 3 decimal places on the slide not on the chart. (Use proper scientific notation if necessary, no E’s, in the equation.) Be able to discuss why you chose this function to model your data based on its characteristics.
   Slide 4: Prediction for five years from now if the trend continues. (Show your equation with independent variable plugged in.) Discuss reasonableness.
   Slide 5–6: Any other pertinent info, your data, URL for links to other sites for additional information, or another data comparison. Include resources used to find data. How your data could help with solutions to particular problems.
2. Present your project to your Algebra II class and to another class. You may not read from the PowerPoint® or from a paper – use index cards to help you present. Dress nicely on presentation day.

**Project Analysis:** Type a discussion concerning what you learned mathematically, historically, and technologically, and express your opinion of how to improve the project.

**Timeline:**
1. Three days from now, bring copy data to class along with a problem statement (why you are examining this data), so it can be approved and you can begin working on it in class. You will hand this in so make a copy.
2. Project is due on

**Final Product:**
1. Disk or flash drive containing the PowerPoint® presentation or email it to me – it should be saved as “your name/s and title of presentation”.
2. A printout of slides in the presentation. (the “Handout” printout, not a page for each slide).
3. Release forms signed by all people in the photographs.
4. Project Analysis
5. Rubric
# Unit 7, Activity 8, Specific Assessment, Modeling to Predict the Future Rubric

<table>
<thead>
<tr>
<th>Written work to be handed in</th>
<th>Teacher Rating</th>
<th>Possible Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data and Problem statement explaining why you are examining this data handed in three days after assigned</td>
<td>xxxxxx</td>
<td>10</td>
</tr>
<tr>
<td>Project Analysis concerning what you learned mathematically, historically, and technologically, and expressing your opinion of how to improve the project</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Printout of PowerPoint® presentation, use of easy–to–read colors and fonts and release forms signed by all people in digital pictures</td>
<td></td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>PowerPoint®</th>
<th>Teacher Rating</th>
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</tr>
</thead>
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<tr>
<td>Slide 1: Introduction of the topic with a relative picture (not clip art), your name, date, class, hour</td>
<td>xxxxxx</td>
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</tr>
<tr>
<td>Slide 2: Statement of the problem, history, and economics of the topic, bullets, not sentences, entering one at a time, no more than 15 words</td>
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<tr>
<td>Slide 3: Scatter plot graph of the data clearly labeled, curve, regression equation, and correlation coefficient. Type regression equation with 3 decimal places on the slide not on the chart. (Use proper scientific notation if necessary, no E’s, in the equation.)</td>
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</thead>
<tbody>
<tr>
<td>Verbal presentation accompanying PowerPoint® (concise, complete, acceptable language, not read from PowerPoint® or paper, dressed nicely)</td>
<td>xxxxxx</td>
<td>10</td>
</tr>
<tr>
<td>Presentation to another class</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Handed in on time</td>
<td></td>
<td>(-10/day late)</td>
</tr>
</tbody>
</table>

**TOTAL** | | 100 |
8.1 **Circle** – write the definition, provide examples of both the standard and graphing forms of the equation of a circle, show how to graph circles, and provide a real-life example in which circles are used.

8.2 **Parabola** – write the definition, give the standard and graphing forms of the equation of a parabola and show how to graph them in both forms, find the vertex from the equation and from the graph, give examples of the equations of both vertical and horizontal parabolas and their graphs, find equations for the directrix and axis of symmetry, identify the focus, and provide real-life examples in which parabolas are used.

8.3 **Ellipse** – write the definition, write standard and graphing forms of the equation of an ellipse and graph both vertical and horizontal, locate and identify foci, vertices, major and minor axes, explain the relationship of $a$, $b$, and $c$, and provide a real-life example in which an ellipse is used.

8.4 **Hyperbola** – write the definition, write the standard and graphing forms of the equation of a hyperbola and draw graph both vertical and horizontal, identify vertices, identify transverse and conjugate axes and provide an example of each, explain the relationships between $a$, $b$, and $c$, find foci and asymptotes, and give a real-life example in which a hyperbola is used.

8.5 **Conic Sections** – define each, explain the derivation of the names, and draw each as a slice from a cone.

8.6 **Degenerate Cases of Conics** – give examples of equations for each and draw the picture representations from cones.
(1) Draw a right triangle with sides 6 and 7 and find the length of the hypotenuse.

(2) Find the distance between the points \((x, y)\) and \((1, 3)\).

(3) Define a circle.
Unit 8, Activity 2, Circles & Lines Discovery Worksheet

Name______________________________ Date________________

Equations & Graphs of Circles

(1) Find the equation of the circle with center (4, 5) passing through the point (–2, 3). Graph the points and the circle.

(2) Find the equation of the circle with the endpoints of the diameter at (2, 6) and (–4, –10). Graph the points and the circle.

(3) Find the equation of the circle with center (4, –3) and tangent to the $x$-axis. Graph the circle.

(4) Find the equation of the circle with center (4, –3) and tangent to the $y$-axis. Graph the circle.

(5) Find the equation of the circle with center (4, –3) and tangent to the line $y = -\frac{3}{4}x$. Graph the line and the circle.

(6) Find the equation of the circle passing through the points (5, 3) and (5, 9) and has a radius = 5. Graph the points and the circle.

(7) On a separate sheet of paper, using the real-world picture you brought in, draw the $x$- and $y$-axes, and find the equation of the circle. Using one of the pictures in your group, write a math story chain in which the first person creates a story problem, which the second person has to solve, and then adds another story problem for the third person to solve, etc.
Unit 8, Activity 2, Circles & Lines Discovery Worksheet with Answers

Name_________________________________________  Date________________________

**Equations & Graphs of Circles**

1. Find the equation of the circle with center (4, 5) passing through the point (–2, 3). Graph the points and the circle.
   \[(x - 4)^2 + (y - 5)^2 = 40\]

2. Find the equation of the circle with the endpoints of the diameter at (2, 6) and (–4, –10). Graph the points and the circle.
   \[(x + 1)^2 + (y + 2)^2 = 73\]

3. Find the equation of the circle with center (4, –3) and tangent to the x-axis. Graph the circle.
   \[(x - 4)^2 + (y + 3)^2 = 9\]

4. Find the equation of the circle with center (4, –3) and tangent to the y-axis. Graph the circle.
   \[(x - 4)^2 + (y + 3)^2 = 16\]

5. Find the equation of the circle with center (4, –3) and tangent to the line \[y = \frac{3}{4}x\]. Graph the line and the circle.
   \[(x - 4)^2 + (y + 3)^2 = 0,\] no circle graph, degenerate case because the center is on the line.

6. Find the equation of the circle passing through the points (5, 3) and (5, 9) and has a radius = 5. Graph the points and the circle.
   \[(x - 1)^2 + (y - 6)^2 = 25\]

7. On a separate sheet of paper, using the real-world picture you brought in, draw the x- and y-axes, and find the equation of the circle. Using one of the pictures in your group, write a math story chain in which the first person creates a story problem, which the second person has to solve, and then adds another story problem for the third person to solve, etc. Answers will vary. See example Circles in the Real World Math Story Chain Example BLM
Student 1: A bee is buzzing around the sunflower and wants to land where the pollen sacs meet the petals. What is the equation of the pollen sacs?

Student 2: If the bee lands on the circular edge of the pollen sacs, 2 units horizontally from the $y$–axis, how far would it be vertically from the $x$–axis?

Student 3: If the bee then takes off from the point found by Student 2 flying perpendicular to the radius of that circle, what is the slope of its flight path?

Student 4: The bee continues on this perpendicular path and ends up at the tip of a petal that is 4 units long (of course sticking straight out from the circular edge of the pollen sacs). How far did it fly?

Solutions:
(1) The equation of the circle is $x^2 + y^2 = 9$.
(2) $2^2 + y^2 = 9$, therefore $y = \sqrt{5}$.
(3) The slope of the radius at that spot is $\frac{\sqrt{5}}{2}$, so the slope of his flight path will be $-\frac{2}{\sqrt{5}}$.
(4) $3^2 + t^2 = 7^2$, so the bee flew $2\sqrt{10}$ units along the line.
**Unit 8, Activity 3, Parabola Discovery Worksheet**

Name_________________________________________ Date________________

I. Vertical Parabolas

1) On the first piece of graph paper, locate the focus at (8, 4) and draw the directrix at y = 2.
2) Use two equal lengths of the string to plot ten points that satisfy the definition of a parabola to create a general parabolic shape. The distance from the focus to a point on the parabola should equal the perpendicular distance from a point on the parabola to the directrix. Locate the vertex on the graph paper using the definition. vertex: ______________________
3) Label one of the points on the parabola \((x, y)\) and the corresponding point on the directrix \((2, y)\). Use the distance formula to find the equation of the parabola. Find the vertex using \(v = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)\). Show work on the back of the graph paper.
   
   equation: ______________________
   vertex: ______________________

4) Complete the square to transform the equation in #3 to graphing form, \(y = a(x - h)^2 + k\). Show work on the back of the graph paper. Find the vertex in this form.
   
   equation: ______________________
   vertex: ______________________

II. Horizontal Parabolas

1) On the second piece of graph paper, locate the focus at (8, 4) and draw the directrix at \(x = 2\).
2) Use two equal lengths of the string to plot ten points that satisfy the definition of a parabola to create a general parabolic shape. The distance from the focus to a point on the parabola should equal the perpendicular distance from a point on the parabola to the directrix. Locate the vertex on the graph paper using the definition. vertex: ______________________
3) Label one of the points on the parabola \((x, y)\) and the corresponding point on the directrix \((2, y)\). Use the distance formula to find the equation of the parabola. Find the vertex using \(v = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)\). Show work on the back of the graph paper.
   
   equation: ______________________
   vertex: ______________________

4) Complete the square to transform the equation in #3 to graphing form, \(x = a(y - k)^2 + h\). Show work on the back of the graph paper. Find the vertex in this form. Enter both of the equations in the graphing calculator to determine if they are coincident.
   
   equation: ______________________
   vertex: ______________________
III. Finding the Focus

(1) What is the distance from the vertex to the focus in Part I? ____ Leading coefficient? ____
What is the distance from the vertex to the focus in Part II? ____ Leading coefficient? ____
How is the distance from the vertex to the focus related to the leading coefficient?

(2) Find the vertex and the focus in the following equations. Graph each by hand after locating two more pairs of points:

(a) \( \frac{1}{16}(x - 2)^2 - 3 \)
(b) \( 2(y + 3)^2 + 5 \)

(c) What effect do changes in the distance to the focus make in the graph?

(3) Graph the following on the graphing calculator and tell how \( a \) affects the shape of the graph:

(a) \( y = 2x^2 + 4x + 5 \)
(b) \( y = -2x^2 + 4x + 5 \)
(c) \( y = 0.5x^2 + 4x + 5 \)

(4) Graph the following horizontal parabolas on a graphing calculator by isolating the \( y \) and graphing the positive and negative radical, then tell how \( a \) affects the shape of the graph:

(a) \( x = 3y^2 - 2 \)
(b) \( x = -3y^2 - 2 \).
Unit 8, Activity 3, Parabola Discovery Worksheet with Answers

Name_________________________________________ Date_____________________

I. Vertical Parabolas

1) On the first piece of graph paper, locate the focus at (8, 4) and draw the directrix at y = 2.
2) Use two equal lengths of the string to plot ten points that satisfy the definition of a parabola to create a general parabolic shape. The distance from the focus to a point on the parabola should equal the perpendicular distance from a point on the parabola to the directrix. Locate the vertex on the graph paper using the definition. vertex: (8, 3)
3) Label one of the points on the parabola (x, y) and the corresponding point on the directrix (x, 2). Use the distance formula to find the equation of the parabola. Find the vertex using
\[
 f\left(\frac{-b}{2a}\right), f\left(\frac{-b}{2a}\right)
\]
Show work on the back of the graph paper.
equation: \(\sqrt{(x - 8)^2 + (y - 4)^2} = y - 2 \Rightarrow y = \frac{1}{4}x^2 - 4x + 19\) vertex: (8, 3)
4) Complete the square to transform the equation in #3 to graphing form, \(y = a(x - h)^2 + k\). Show work on the back of the graph paper. Find the vertex in this form. Enter both of the equations in calculator to determine if they are coincident.
equation: \(y = \frac{1}{4}(x - 8)^2 + 3\) vertex: (8, 3)

II. Horizontal Parabolas

1) On the second piece of graph paper, locate the focus at (8, 4) and draw the directrix at x = 2.
2) Use two equal lengths of the string to plot ten points that satisfy the definition of a parabola to create a general parabolic shape. The distance from the focus to a point on the parabola should equal the perpendicular distance from a point on the parabola to the directrix. Locate the vertex on the graph paper using the definition. vertex: (5, 4)
3) Label one of the points on the parabola (x, y) and the corresponding point on the directrix (2, y). Use the distance formula to find the equation of the parabola. Find the vertex using
\[
 f\left(\frac{-b}{2a}\right), -b
\]
Show work on the back of the graph paper.
equation: \(\sqrt{(x - 8)^2 + (y - 4)^2} = x - 2 \Rightarrow x = \frac{1}{12}y^2 - \frac{2}{3}y + \frac{19}{3}\) vertex: (5, 4)
4) Complete the square to transform the equation in #3 to graphing form, \(x = a(y - k)^2 + h\). Show work on the back of the graph paper. Find the vertex in this form.
equation: \(y = \frac{1}{12}(y - 4)^2 + 5\) vertex: (5, 4)
III. Finding the Focus

(1) What is the distance from the vertex to the focus in Part I? \( \frac{1}{4} \)  Leading coefficient? \( \frac{1}{4} \)

What is the distance from the vertex to the focus in Part II? \( \frac{3}{12} \)  Leading coefficient? \( \frac{1}{12} \)

How is the distance from the vertex to the focus related to the leading coefficient?

The leading coefficient as formed by \( \frac{1}{4c} \) where \( c \) is the distance from the vertex to the focus.

(2) Find the vertex and the focus in the following equations. Graph each by hand after locating two more pairs of points:

(a) \[ y = \frac{1}{10}(x - 2)^2 - 3 \]
vertex: \( (2, -3) \)  focus: \( (2, 1) \)

(b) \[ x = 2(y + 3)^2 + 5 \]
vertex: \( (5, -3) \)  focus: \( \left( \frac{41}{8}, -3 \right) \)

(c) What effect do changes in the distance to the focus make in the graph?
The closer the focus is to the vertex, the narrower the graph.

(3) Graph the following on the graphing calculator and tell how \( a \) affects the shape of the graph:

(a) \[ y = 2x^2 + 4x + 5 \]
(b) \[ y = -2x^2 + 4x + 5 \]
(c) \[ y = 0.5x^2 + 4x + 5 \]

For vertical parabolas, the positive value for “\( a \)” makes the graph open up while the negative value for “\( a \)” makes the graph open down. The smaller the value of “\( a \)”, the wider the parabola.

(4) Graph the following horizontal parabolas on a graphing calculator by isolating the \( y \) and graphing the positive and negative radical, then tell how \( a \) affects the shape of the graph:

(a) \[ x = 3y^2 - 2 \]
(b) \[ x = -3y^2 - 2 \]

For horizontal parabolas, the positive “\( a \)” makes the graph open to the right and the negative “\( a \)” makes the graph open to the left.
**Unit 8, Activity 4, Ellipse Discovery Worksheet**

Name__________________________________________ Date________________

**Drawing an Ellipse**

ellipse ≡ set of all points in plane in which the sum of the focal radii is constant.

(1) Use the definition of ellipse to sketch its graph by sticking the pins in the ends of the string and holding them at the given points (called the foci). These will be to be two of the vertices of a triangle. Place a pencil in the third vertex of the triangle. Move the pencil around the foci top and bottom.

(2) Since the string represents the sum of the focal radius, what is the length of the string using units on the graph paper? ______________________________

(3) Draw the longest axis of symmetry (the major axis). What is its length? __________

How does this relate to the sum of the focal radii?

(4) Draw an isosceles triangle with the base on the major axis, the vertices of the base at the foci, and the third vertex the end of the shorter axis of symmetry (the minor axis).

What is the length of the altitude which is ½ the minor axis? __________________________

What is the length of the legs of the isosceles triangle? ________________________________

(5) Label ½ the major axis as \(a\), ½ the minor axis as \(b\), and the distance from the center of the ellipse to the focus as \(c\).

How long is each? \(a = \) _______ , \(b = \) _______ , \(c = \) _______

(6) What is the relationship of the length of the legs of the isosceles triangle to the length of ½ the major axis?

(7) What is the relationship between \(a\), \(b\), and \(c\)?

(8) Look at the back of the cardboard to find the equation of the ellipse. How is the information found related to this equation?

(9) Tape the graph to the board and write the equation below it.

Blackline Masters, Algebra II  
Louisiana Comprehensive Curriculum, Revised 2008
**Drawing an Ellipse**

**ellipse** ≡ set of all points in plane in which the sum of the focal radii is constant.

1. Use the definition of ellipse to sketch its graph by sticking the pins in the ends of the string and holding them at the given points (called the foci). These will be two of the vertices of a triangle. Place a pencil in the third vertex of the triangle. Move the pencil around the foci top and bottom.

2. Since the string represents the sum of the focal radius, what is the length of the string using units on the graph paper? *answers will vary*

3. Draw the longest axis of symmetry (the major axis). What is its length? *answers will vary*

   **How does this relate to the sum of the focal radii?** The length of the major axis equals the sum of the focal radii.

4. Draw an isosceles triangle with the base on the major axis, the vertices of the base at the foci, and the third vertex the end of the shorter axis of symmetry (the minor axis).

   **What is the length of the altitude which is ½ the minor axis?** *answers will vary*

   **What is the length of the legs of the isosceles triangle?** *answers will vary*

5. Label ½ the major axis as $a$, ½ the minor axis as $b$, and the distance from the center of the ellipse to the focus as $c$.

   **How long is each?** *answers will vary* $a = \ldots , b = \ldots , c = \ldots$

6. What is the relationship of the length of the legs of the isosceles triangle to the length of ½ the major axis? The leg of the isosceles triangle equals half the major axis.

7. What is the relationship between $a$, $b$, and $c$? $b^2 + c^2 = a^2$

8. Look at the back of the cardboard to find the equation of the ellipse. How is the information found related to this equation? The square root of the denominators equals half the lengths of the major and minor axes. If the major axis is horizontal, then half the major axis squared is under the $x^2$. If the major axis is vertical, then half the major axis squared is under the $y^2$.

9. Tape the graph to the board and write the equation below it.
Saga of the Roaming Ellipse

You are an ellipse.

Your owner is an Algebra II student who moves you and stretches you.

Using all you know about yourself, describe what is happening to you while the Algebra II student is doing his/her homework.

You must include ten facts or properties of an ellipse in your discussion.

(Write in paragraph form but number the ten facts.)
Saga of the Roaming Hyperbola

You are a hyperbola. Your owner is an Algebra II student who moves you and stretches you. Using all you know about yourself, describe what is happening to you while the Algebra II student is doing his/her homework. You must include ten facts or properties of a hyperbola in your discussion. (Write in paragraph form but number the ten facts.)
Saga of the Roaming Parabola

You are a parabola. Your owner is an Algebra II student who moves you and stretches you. Using all you know about yourself, describe what is happening to you while the Algebra II student is doing his/her homework. You must include ten facts or properties of a parabola in your discussion. (Write in paragraph form but number the ten facts.)
Graph the following equations by hand on the graph below.

<table>
<thead>
<tr>
<th>EQUATIONS</th>
<th>RESTRICTIONS on DOMAINS (x)/RANGES (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = -\frac{1}{9}(x - 6)^2 + 14$</td>
<td>D: [0, 12]</td>
</tr>
<tr>
<td>$y = -\frac{1}{9}x^2 - \frac{4}{3}x + 10$</td>
<td>D: [-12, 0]</td>
</tr>
<tr>
<td>$y = -2x + 34$</td>
<td>D: [12, 13]</td>
</tr>
<tr>
<td>$2x - y = -34$</td>
<td>D: [-13, -12]</td>
</tr>
<tr>
<td>$x = 13 \text{ or } x = -13$</td>
<td>R: [5, 8]</td>
</tr>
<tr>
<td>$y = \frac{21}{13}x - 16$</td>
<td>D: [-13, 13]</td>
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<td>D: [-12, 0]</td>
</tr>
<tr>
<td>3 $y = -2x + 34$</td>
<td>D: [12, 13]</td>
</tr>
<tr>
<td>4 $2x - y = -34$</td>
<td>D: [-13, -12]</td>
</tr>
<tr>
<td>5 $x = 13$ or $x = -13$</td>
<td>R: [5, 8]</td>
</tr>
<tr>
<td>6 $y = \frac{21}{13}x - 16$</td>
<td>D: [-13, 13]</td>
</tr>
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</table>
Unit 8, Activity 10, Graphing Art Sailboat Graph

Directions
1. Find at least one equation for each section of the graph from 1 through 11.
2. Use three different forms of equations for sections 6, 7, and 8.
3. Use domain or range restrictions and compound statements using “and” and “or” where appropriate.

From the Mathematics Teacher, February 1995
Unit 8, Activity 10, Graphing Art Sailboat Equations

<table>
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<tbody>
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<td>11</td>
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### Unit 8, Activity 10, Graphing Art Sailboat Equations with Answers

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>1 (x = -2)</td>
<td>R: ([-8, -1] \cup [13, 16])</td>
</tr>
<tr>
<td>2 (y = 15) or (y = 16)</td>
<td>D: ([-2, 0])</td>
</tr>
<tr>
<td>3 (x = 0)</td>
<td>D: ([15, 16])</td>
</tr>
<tr>
<td>4 (y = -\frac{7}{5}</td>
<td>x + 2</td>
</tr>
<tr>
<td>5 (x = 2(y - 10)^2 - 3)</td>
<td>R: ([9, 11])</td>
</tr>
<tr>
<td>6 (y = -\frac{1}{3}x + 7)</td>
<td>D: ([-5.077, 3])</td>
</tr>
<tr>
<td>7 (y - 6 = -\frac{1}{3}(x + 3))</td>
<td>D: ([-6.231, 4.875])</td>
</tr>
<tr>
<td>8 (x + 3y = 10)</td>
<td>D: ([-7.192, 6.438])</td>
</tr>
<tr>
<td>9 (y = \frac{1}{18}(x + 2)^2 - 16)</td>
<td>D: ([-14, 10])</td>
</tr>
<tr>
<td>10 (y = -1)</td>
<td>D: ([-12, 8])</td>
</tr>
<tr>
<td>11 (y = -8)</td>
<td>D: ([-14, 10])</td>
</tr>
</tbody>
</table>
Unit 8, Activity 10, Graphing Art Project Directions

Directions

1. Using graph paper, draw a picture containing graphs of functions we have studied this year. The picture must contain graphs of a minimum of ten different equations with at least the following:
   - two lines
   - two parabolas (one vertical and one horizontal)
   - one absolute value function
   - two of the following – square root, cube root, greatest integer, cubic, exponential, or logarithm functions
   - one circle or portion thereof
   - one ellipse or portion thereof
   - one hyperbola or portion thereof

2. Domains and ranges may be determined by solving equations simultaneously or by using the graphing calculator to graph the function and find the point of intersection. If these points are not integers, round them three places behind the decimal.

3. Make two copies of your graph. Hand in a rough copy of your final figure with the different parts of your graph numbered to coincide with the equations. Later you will hand in an unnumbered final copy of your picture that has been outlined in a black fine-tip marker. Hand in the equations neatly printed in black ink on the Graphing Art Project Equations BLM or typed using MathType® or EquationWriter®.

Evaluation

Attached is the Graphing Art Evaluation Rubric that both you and I will use to evaluate the project. Fill in the sheet and hand in with your final copy.

Time Table

1. A rough draft picture with at least ten of the equations and domain/range restrictions is due on ________________.

2. The final picture, all equations in proper form, and the completed evaluation sheet are due on ________________.
Unit 8, Activity 10, Graphing Art Graph Paper

Name______________________________ Date__________________

[Graph paper with grid]
### Unit 8, Activity 10, Graphing Art Project Equations

Mathematician/Artist: ___________________ Title: ___________________

<table>
<thead>
<tr>
<th>EQUATIONS</th>
<th>RESTRICTIONS on DOMAINS (x) / RANGES (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<td>20</td>
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</tbody>
</table>
### Unit 8, Activity 10, Graphing Art Project Equations – Page 2

Mathematician/Artist: ____________________  Title: ____________________

<table>
<thead>
<tr>
<th>EQUATIONS</th>
<th>RESTRICTIONS on DOMAINS (x) / RANGES (y)</th>
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<tbody>
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### Grading Rubric

<table>
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<tr>
<th>Types of graph (minimum # required)</th>
<th># correct</th>
<th>total #</th>
<th>% correct</th>
<th>weighted score</th>
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</thead>
<tbody>
<tr>
<td>Lines (2)</td>
<td></td>
<td></td>
<td>% x 10 =</td>
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<tr>
<td>Parabolas (2 – one horizontal and one vertical)</td>
<td></td>
<td></td>
<td>% x 10 =</td>
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<tr>
<td>Absolute value (1)</td>
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<td>% x 10 =</td>
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<tr>
<td>Other (see list) (2)</td>
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<td>% x 20 =</td>
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<tr>
<td>Circle (1)</td>
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<td>% x 10 =</td>
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<tr>
<td>Ellipse (1)</td>
<td></td>
<td></td>
<td>% x 10 =</td>
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<tr>
<td>Hyperbola (1)</td>
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<td></td>
<td>% x 10 =</td>
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<tr>
<td>Domain/Range Restrictions</td>
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<td>% x 10 =</td>
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<tr>
<td>Complexity Rating</td>
<td>Rating 1 – 10 (see rubric below)</td>
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<tr>
<td>Format Rating</td>
<td>Rating 1 – 5 (see teacher comments below)</td>
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</table>

### Complexity Rubric

A maximum of 10 points can be earned in any of the following ways:

<table>
<thead>
<tr>
<th>Complexity</th>
<th>0 points</th>
<th>1 point</th>
<th>2 points</th>
<th>3 points</th>
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</thead>
<tbody>
<tr>
<td># of equations used</td>
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<tr>
<td># of complex domains/ranges</td>
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<tr>
<td># of equations in different forms</td>
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<tr>
<td># of square root, cube root, greatest integer, cubic, exponential, or logarithmic functions (2 required)</td>
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</tbody>
</table>

Rating: 1 – 10

### Format Score

This score is a rating of 1 – 5 based on readability, proper labeling, number of equations in rough draft, outlined final draft, neat equations and final picture, etc.

Rating: _______ Teacher comments on format: __________________________________________